FUNDAMENTAL GRAPHICS FOR THREE DIMENSIONAL ORBITS.

by

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ABSTRACT

Orbital theory with the spherical polar coordinates is used to show that the orbit is not in general an ellipse, despite the fact that angular momentum is rigorously conserved. Kepler's second law is changed from the equivalent in planar orbital theory and the time taken to complete an orbit is also changed. It is shown that angular momentum is conserved for all θ of the (r, θ , ϕ) system, so conservation of angular momentum does not mean that the orbit is a planar ellipse in r and ϕ . The three dimensional orbit is characterized by sets of graphics which show an intricate structure.

Keywords: ECE theory, three dimensional orbit theory, graphics, orbital type structures.

UFT 272

1. INTRODUCTION

In recent papers of this series {1 - 10} the theory of orbits has been developed for the first time in three dimensions using spherical polar coordinates {11, 12} rather than plane polar coordinates as is traditional. It has been shown that conservation of angular momentum does not imply that the orbit is a planar ellipse. That is the result of a circular argument in which the orbit is assumed to be planar and then shown to be planar. In the correct theory the orbit must be calculated in three dimensions using the spherical polar coordinates. An entirely different theory emerges full of intricate structure. In general the orbit is not planar and r is a function both of θ and ϕ, where the spherical polar system is (r, θ, ϕ). In Section 2 the three dimensional theory is developed to the point where various direct comparisons can be made with the two dimensional theory. For example it is shown that in three dimensions the function of r against ϕ is not an ellipse in general, it approaches a precessing ellipse and becomes an exact ellipse if and only if the angular momentum has only one Cartesian component, in the Z axis. In general in three dimensions the angular momentum must have three components, X, Y, and Z.

It is shown that Kepler's second law is changed when a rigorously three dimensional theory is used, and the time taken to complete one orbit is also changed. In the three dimensional theory, conservation of angular momentum is shown to be true for all θ showing in a clear way that conservation of angular momentum does not imply that θ is fixed at $\pi/2$, as in the traditional planar theory which has been accepted for over four hundred years. Various sets of equations are developed for graphics that show the difference between the three dimensional and two dimensional orbits directly. These graphics are known from the preceding paper to show an intricate orbital like structure.

In Section 3 the set of graphics is summarized with a few examples which are enough to show that three dimensional orbit theory is far richer in structure and information than the traditional, four hundred year old, planar theory.

2. DIRECT COMPARISONS OF THREE AND TWO DIMENSIONAL ORBIT THEORY

In three dimensional orbits:

$$\underline{L} = \underline{r} \times \underline{p} \qquad -(1)$$

where $\underline{L}$ is the total angular momentum, $\underline{r}$ the position vector and $\underline{p}$ the linear momentum vector. The conservation of angular momentum means that:

$$\frac{d\underline{L}}{dt} = \underline{0} \qquad -(2)$$

where:

$$\underline{p} = m\frac{d\underline{r}}{dt}. \qquad -(3)$$

Therefore:

$$\frac{d\underline{L}}{dt} = \dot{\underline{r}} \times \underline{p} + \underline{r} \times \dot{\underline{p}} \qquad -(4)$$

where the force is:

$$\underline{F} = \dot{\underline{p}} = m\underline{a}. \qquad -(5)$$

As shown in the preceding paper:

$$\underline{a} = \ddot{\underline{r}} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{\underline{\omega}} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt}\underline{e}_r \qquad -(6)$$

and for a an inverse square law of attraction:

$$\underline{F} = -\frac{k}{r^2}\underline{e}_r \qquad -(7)$$

the following equation is true:

$$\dot{\underline{\omega}} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt}\underline{e}_r = \underline{0}. \qquad -(8)$$

Therefore the acceleration is:

$$\underline{a} = \ddot{\underline{r}} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \left(\ddot{r} - \left(r\dot{\theta}^2 + r\sin^2\theta\,\dot{\phi}^2\right)\right)\underline{e}_r \quad -(9)$$

and is parallel to the position vector:

$$\underline{r} = r\,\underline{e}_r \quad -(10)$$

So

$$\underline{r} \times \underline{F} = \underline{0} \quad -(11)$$

QED.

Angular momentum is conserved for all θ , not just for

$$\theta = \frac{\pi}{2} \quad -(12)$$

as in the traditional, four hundred year old, theory. This result is true for any force law between m and M directed along $\underline{e}_r$ in a three dimensional space. The angular momentum $\underline{L}$ is not confined to the Z axis, and both $\underline{r}$ and $\underline{p}$ are three dimensional. The usual theory of orbits asserts without proof that $\dot{\theta}$ in Eq. (9) is zero and that θ is always $\pi/2$.

The three dimensional beta orbit as defined in the preceding paper is:

$$r = \frac{\alpha}{1 + \epsilon\cos\beta} \quad -(13)$$

where β can be expressed in terms of ϕ as follows:

$$\tan\beta = \frac{L}{L_Z}\tan\phi. \quad -(14)$$

Here L is the magnitude of the total angular momentum and L_Z is its component in the Z axis. The half right latitude α and the eccentricity ϵ can be expressed in terms of L and the total energy E as in UFT271. It follows that:

$$\cos^2\beta = \cos^2\phi \left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi\right)^{-1} \quad -(15)$$

from which it is clear that as:

$$L \rightarrow L_Z \quad -(16)$$

then the two dimensional limit is obtained self consistently:

$$\beta \rightarrow \phi. \quad -(17)$$

Now define:

$$\cos(x\phi) = \cos\phi \left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi\right)^{-1} \quad -(18)$$

and a precession factor can be defined as:

$$x = \frac{1}{\phi} \cos^{-1}\left(\cos\phi \left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi\right)^{-1/2}\right) \quad -(19)$$

to give the precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(x\phi)} \quad -(20)$$

It is observed experimentally that:

$$x = 1 + \frac{3MG}{\alpha c^2}. \quad -(21)$$

So Eqs. (19) and (21) define an angle ϕ_0.

Therefore three dimensional orbits have evolved to precessing two dimensional orbits. In general:

$$r = \frac{\alpha}{1+\epsilon\cos\beta} \qquad -(22)$$

where:

$$\cos\beta = \cos\phi\left(\cos^2\phi + \left(\frac{L_X}{L_Z}\right)^2 \sin^2\phi\right)^{-1/2} \qquad -(23)$$

It is clear that Eq. (22) is no longer an ellipse and primordial three dimensional orbits evolve into observed two dimensional precessing orbits.

The time dependence of this process is given by:

$$\frac{d\beta}{dt} = \frac{L}{mr^2} \qquad -(24)$$

so:

$$dt = \frac{mr^2}{L}\, d\beta \qquad -(25)$$

and

$$t = \frac{m\alpha^2}{L}\int \frac{d\beta}{(1+\epsilon\cos\beta)^2} \qquad -(26)$$

so t can be evaluated in terms of L and of E . This procedure can be used for an animation.

As shown in the preceding paper UFT271 the complete set of equations for three dimensional orbits include the following:

$$\ddot{r} = r\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) - MG/r^2 \qquad -(27)$$

$$r\ddot{\theta} - r\dot{\phi}^2 \sin\theta\cos\theta + 2\dot{r}\dot{\theta} = 0 \qquad -(28)$$

$$r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + 2\dot{r}\dot{\phi}\sin\theta = 0 \qquad -(29)$$

where M is the attracting mass M and where G is Newton's constant. These three equations are augmented by the following set of equations:

$$\dot{\phi} = \frac{L_Z}{mr^2 \sin^2\theta} \quad -(30)$$

$$\dot{\theta} = \frac{1}{mr^2}\left(L^2 - \frac{L_Z^2}{\sin^2\theta}\right)^{1/2} \quad -(31)$$

$$\sin^2\theta = \left(\frac{L_Z}{L}\right)^2 + \left(1 - \left(\frac{L_Z}{L}\right)^2\right)\left(\frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi}\right) \quad -(32)$$

$$= 1 - \cos^2\theta$$

$$\dot{r} = \left(\frac{2}{m}\left(E - \frac{L^2}{2mr^2} + \frac{k}{r}\right)\right)^{1/2} \quad -(33)$$

$$r = \frac{\alpha}{1 + \epsilon \cos\beta} \quad -(34)$$

$$\cos\beta = \cos\phi \left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi\right)^{-1/2} \quad -(35)$$

$$\cos\beta = \left(1 - \frac{L^2 \cos^2\theta}{L^2 - L_Z^2}\right)^{1/2} \quad -(36)$$

$$E = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\beta}^2\right) - \frac{k}{r}. \quad -(37)$$

In UFT271 it was shown that this intricate and interlinked set of equations leads to orbital like structure in orbit theory. In this section they are used in various ways to more fully characterize three dimensional orbits as follows.

1) The function $\ddot{r}$ can be expressed in terms , ϕ from the three dimensional theory and compared directly with the results from two dimensional theory:

$$\ddot{r} = r\dot{\phi}^2 - MG/r \quad -(38)$$

$$\dot{\phi} = \frac{L}{mr^2}, \quad -(39)$$

$$r = \alpha / (1+\epsilon\cos\phi). \quad -(40)$$

2) The function $\ddot{\phi}$ can be expressed in terms of ϕ without involving the tangent function as was used in UFT271. From Eq. (29):

$$\ddot{\phi} = -2\dot{\phi}\left(\dot{\theta}\frac{\cos\theta}{\sin\theta} + \frac{\dot{r}}{r}\right) \quad -(41)$$

which can be worked out over the complete range of ϕ to give the complete orbital structure without the restrictions imposed by the tangent function by using Eq. (32) to define the $\sin\theta$ and $\cos\theta$ functions. In two dimensions:

$$\ddot{\phi} = -2\dot{\phi}\frac{\dot{r}}{r}. \quad -(42)$$

3) The graph of $\ddot{r}$ versus θ can be constructed using Eqs. (27), (30), (34) and (36) with the following definition of β in terms of θ :

$$\cos^2\beta = 1 - \frac{L^2\cos^2\theta}{L^2 - L_Z^2}. \quad -(43)$$

4) The graph of $\ddot{\theta}$ against θ can be constructed using Eq. (28) written as follows:

$$\ddot{\theta} = \dot{\phi}^2\sin\theta\cos\theta - \frac{2\dot{r}\dot{\theta}}{r} \quad -(44)$$

and using Eqs. (30), (34), (36), (31) and (33). In two dimensional orbit theory there is no graph of $\ddot{\theta}$ against θ because the latter is fixed at $\pi/2$.

5) It is also possible to graph $\ddot{r}$ as a function of both θ and ϕ . This is a three

dimensional polar graph in which the three dimensional theory gives intricate results. In order to produce this kind of graph $\dot{\phi}$ is expressed in terms of θ as follows

$$\dot{\phi} = \frac{L_Z}{mr^2 \sin^2\theta} \quad -(45)$$

and $\dot{\theta}$ is expressed in terms of ϕ as follows:

$$\dot{\theta} = \left(L^2 - L_Z^2 / \sin^2\theta\right)^{1/2} / (mr^2) - (46)$$

$$\sin^2\theta = \left(\frac{L_Z}{L}\right)^2 + \left(1 - \left(\frac{L_Z}{L}\right)^2\right) \cos^2\phi \left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi\right)^{-1} \quad -(47)$$

There are at least four types of graph of r($\ddot{\phi}$, θ) defined as follows:

a) Type one is produced by use of Eq. (47) in Eq. (45) and Eq. (36) in Eq. (34).

b) Type two is produced by use of Eq. (47) in Eq. (46) and Eq. (36) in Eq. (34).

c) Type three is produced by use of Eq. (47) in Eq. (45) and Eq. (35) in Eq. (34).

d) Type four is produced by use of Eq. (47) in Eq. (46) and Eq. (35) in Eq. (34).

The equivalent from two dimensional orbital theory is:

$$\ddot{r} = r\dot{\phi}^2 - MG/r^2 - (48)$$

$$\dot{\phi} = L_Z / (mr^2) \quad - (49)$$

$$r = \frac{\alpha}{1 + \epsilon \cos\phi} \quad - (50)$$

6) The function $\ddot{\phi}$ as a function of both ϕ and θ can be constructed from Eq. (29) written as:

$$\ddot{\phi} = -2\dot{\phi}\left(\dot{\theta}\frac{\cos\theta}{\sin\theta} + \frac{\dot{r}}{r}\right) - (51)$$

with the auxiliary set of equations already defined as above. This produces a large number of possibilities, some of which are defined as follows.

a) Type one is defined by expressing $\dot{\phi}$ in terms of θ using Eq. (45) and $\dot{r}/r$

in terms of ϕ using Eqs. (33) to (35). In this procedure $\dot{\theta}$ is expressed in terms of θ using Eq. (46).

b) Type two is defined by expressing $\dot{\phi}$ in terms of ϕ using Eqs. (45) and (47),. and expressing $\dot{r}/r$ in terms of θ using Eqs. (34), (35) and (32). In this procedure $\dot{\theta}$ is expressed in terms of θ using Eq. (46).

c) The procedure leading to type one is repeated but $\dot{\theta}$ is expressed in terms of ϕ using Eqs. (46) and (47).

d) The procedure leading to type two is repeated but $\dot{\theta}$ is expressed in terms of ϕ using Eqs. (46) and (47).

Types five to eight can be defined in the same way as types one to four, but expressing $\cos\theta \mid \sin\theta$ in terms of ϕ using Eq. (47) and

$$\cos^2\theta = 1 - \sin^2\theta. \quad -(52)$$

There are many more permutations and combinations possible which all characterize the three dimensional theory of orbits.

7) Finally the graph of $\ddot{\theta}$ versus ϕ and θ can be defined by:

$$\ddot{\theta} = \dot{\phi}^2 \sin\theta\cos\theta - 2\frac{\dot{r}\dot{\theta}}{r}. \quad -(53)$$

a) Type one is defined by expressing $\dot{\phi}$ in terms of ϕ Using Eqs. (45) and (47), and by expressing $2\frac{\dot{r}\dot{\theta}}{r}$ in terms of θ using Eqs. (33), (31), (34), (35), and (32).

b) Type two is defined by expressing $\dot{\phi}$ in terms of θ using Eq. (45) and by expressing $2\frac{\dot{r}\dot{\theta}}{r}$ in terms of ϕ using Eqs. (34), (33), (31) and (32).

There are many other possibilities like this.

3. GRAPHICS AND DISCUSSION

Section by Dr. Horst Eckardt.

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