The Complete Equations of Classical Dynamics in ECE Theory

by

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Abstract

The complete equations of classical dynamics in ECE theory are deduced directly from Cartan geometry, the first Cartan structure equation, the Bianchi identity, and the Hodge dual of the Bianchi identity. These equations are fully relativistic and reduce to the well known non-relativistic limits when the spin connection becomes very small. The equations of ECE dynamics have the same structure as the equations of electrodynamics in ECE unified field theory, and the ECE equations of dynamics generalize the gravito-magnetic equations of the standard model to fully relativistic situations without any assumption on linearization.

Keywords: ECE equations of relativistic classical dynamics, reduction to non-relativistic dynamics.

20.1 Introduction

The fundamental idea of relativity is that physics is an objective subject, the observation of nature and its explanation without anthropomorphic distortion. Geometry is one method of representing objectivity: it is well known that special relativity uses the Minkowski space-time, and that Einstein's approach to objectivity (general relativity) used Riemann geometry. The use of Riemann geometry by Einstein and Hilbert in 1915 led to the well known Einstein-Hilbert (EH) field equation. The latter reduces to Newtonian dynamics in the weak field limit. The latter can be thought of as the approach to

Minkowski space-time. Recently, during the development of Einstein Cartan Evans (ECE) field theory [1–10], it has been realized that the Einsteinian geometry is severely self-inconsistent because of its neglect of space-time torsion. This omission means that the Einsteinian geometry does not obey the Bianchi identity of Cartan geometry and also its Hodge dual identity. This means that the Einstein field equation is obsolete, no physical inference can be drawn from an incorrect geometry in relativity theory because objectivity has been incorrectly represented.

In Section 20.2 the geometrically correct equations of relativistic dynamics are inferred directly from the correct Cartan geometry, space-time torsion playing its proper central role. There are two basic equation structures: based on orbital torsion and spin torsion. The equation based on orbital torsion for example is the dynamical analogue of the Coulomb inverse square law of ECE electrodynamics [1–10], and in the non-relativistic limit gives the Newton inverse square law. This equation of orbital torsion has its Hodge dual equation, and the two equations together control the dynamics. Similarly there is a dynamical equation of spin torsion which is the direct analogue of the Ampere Maxwell law of ECE electrodynamics. This spin torsion equation again has a Hodge dual spin torsion equation, the dynamical analogue of the Faraday law of induction of ECE electrodynamics. Finally in Section 20.2 the generally relativistic Euler type equation fo motion is obtained from the first Cartan structure equation, which defines the Cartan torsion as the covariant exterior derivative of the Cartan tetrad. The Euler equation is regained in the limit of very small, but non-zero, spin connection. The concept of force field in classical dynamics is inferred to be orbital space-time torsion multiplied by rest energy. The concept of torque field in classical dynamics is inferred to be the integral over distance of the spin torsion multiplied by rest energy. These equations of relativistic dynamics are fully compatible with the Bianchi identity and the dual identity as required by objectivity. Orbits are not due to the EH field equation, that obsolete inference was an anthropomorphic distortion introduced by the neglect of the central feature of both dynamics and electrodynamics-space-time torsion.

20.2 The Equations of Relativistic Dynamics

The basic structure of the equations is given in the standard notation [11] of differential geometry by the first Bianchi identity:

$$D \wedge T^a := R^a{}_b \wedge q^b \tag{20.1}$$

and its Hodge dual identity:

$$D \wedge \widetilde{T}^a := \widetilde{R}^a{}_b \wedge q^b \tag{20.2}$$

where $D\wedge$ denotes the covariant exterior derivative [1–11], T^a the space-time torsion form, R^a_b the space-time curvature form, and q^b the space-time tetrad form. The symbol $\wedge$ denotes the wedge product of differential geometry [1–11] and the tilde symbol denotes the Hodge dual transform in four dimensions. In tensorial notation Eq. (20.1) can be reduced to [1–10]:

$$D_\mu \widetilde{T}^{\kappa\mu\nu} = \widetilde{R}^\kappa{}_\mu{}^{\mu\nu} \tag{20.3}$$

and Eq. (20.2) can be reduced to:

$$D_\mu T^{\kappa\mu\nu} = R^\kappa{}_\mu{}^{\mu\nu}. \tag{20.4}$$

Eq. (20.3) is the homogeneous equation of Cartan geometry, and Eq. (20.4) is the inhomogeneous equation of Cartan geometry. Each tensorial equation can be developed as two vectorial equations. It is convenient to rewrite the equations as:

$$\partial_\mu \widetilde{T}^{\kappa\mu\nu} = \widetilde{J}^{\kappa\nu} \tag{20.5}$$

and

$$\partial_\mu T^{\kappa\mu\nu} = J^{\kappa\nu} \tag{20.6}$$

where the terms on the right hand side subsume the spin connection and are current terms in analogy with ECE electrodynamics [1–11]. Eqs. (20.5) and (20.6) are similar to the well known Maxwell Heaviside (MH) field equations:

$$\partial_\mu \widetilde{F}^{\mu\nu} = 0 \tag{20.7}$$

and

$$\partial_\mu F^{\mu\nu} = J^\nu / \epsilon_0 \tag{20.8}$$

where $F^{\mu\nu}$ is the electromagnetic field tensor, $\widetilde{F}^{\mu\nu}$ is its Hodge dual, J^ν is charge current density and ϵ_0 is the vacuum permittivity in S.I. units. However, Eqs. (20.5) and (20.6) are ones of general relativity, written in a four dimensional, dynamic, space-time with torsion and curvature both present. The MH equations are written in the Minkowski space-time of special relativity, the flat and static space-time with no torsion and no curvature.

The laws of orbital torsion are obtained with the index choice:

$$\kappa = 0 \tag{20.9}$$

giving:

$$\partial_\mu \widetilde{T}^{0\mu\nu} = \widetilde{J}^{0\nu} \qquad (20.10)$$

and

$$\partial_\mu T^{0\mu\nu} = J^{0\nu}. \qquad (20.11)$$

In vector notation these are respectively:

$$\boldsymbol{\nabla} \cdot \widetilde{\boldsymbol{T}} = \widetilde{J}^0 \qquad (20.12)$$

and

$$\boldsymbol{\nabla} \cdot \boldsymbol{T} = J^0. \qquad (20.13)$$

These are respectively the dynamical analogues of the ECE Gauss law:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \qquad (20.14)$$

and the ECE Coulomb law

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho/\epsilon_0 \qquad (20.15)$$

where $\boldsymbol{B}$ is magnetic flux density, $\boldsymbol{E}$ is electric field strength, and ρ is charge density.

In Cartesian coordinates the orbital torsion vector is [1–10]:

$$\boldsymbol{T} = T^{010}\boldsymbol{i} + T^{020}\boldsymbol{j} + T^{030}\boldsymbol{k} \qquad (20.16)$$

and its Hodge dual is:

$$\widetilde{\boldsymbol{T}} = \widetilde{T}^{010}\boldsymbol{i} + \widetilde{T}^{020}\boldsymbol{j} + \widetilde{T}^{030}\boldsymbol{k}. \qquad (20.17)$$

From the well known experimental fact that both the Coulomb and Newton laws are inverse square laws, Eq. (20.13) is identified as giving the Newton inverse square law in the non-relativistic limit. The latter is defined as the limit in which the spin connection becomes very small, i.e. the limit in which Minkowski space-time is approached. In this limit the torsion and curvature approach zero, but are still finite. By analogy between electric field strength $\boldsymbol{E}$ and acceleration due to gravity $\boldsymbol{g}$ it may be inferred that:

$$\boldsymbol{g} = c^2\boldsymbol{T}. \qquad (20.18)$$

This inference is an example of the basic ECE hypothesis [1–10]:

$$A^a = A^{(0)} q^a \tag{20.19}$$

where A^a is the electromagnetic potential form and where $cA^{(0)}$ is a primordial or fundamental voltage proportional to the charge e on the proton. The ECE hypothesis turns geometry into physics.

Therefore the fundamental relativistic equations of motion that reduce to Newtonian dynamics are:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 J^0 \tag{20.20}$$

and its Hodge dual:

$$\boldsymbol{\nabla} \cdot \widetilde{\boldsymbol{g}} = c^2 \widetilde{J}^0. \tag{20.21}$$

From analogy with electrodynamics the Hodge dual current is zero for all practical purposes, and this analogy with electrodynamics comes from the fundamental hypothesis (20.19). Therefore $\widetilde{\boldsymbol{g}}$ is the dynamical analogue of the magnetic flux density in the ECE Gauss law of magnetism.

In the non-relativistic limit the spin connection becomes very small, so:

$$J^0 \rightarrow R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30} \tag{20.22}$$

where the right hand side is a sum of curvature elements. The mass density (kilograms per cubic meter) is identified from Eqs. (20.20) and (20.22) as:

$$\rho_m = \frac{1}{k}(R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30}) \tag{20.23}$$

where k is the Einstein constant in meters per kilogram. Therefore in the non-relativistic limit:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 k \rho_m \tag{20.24}$$

which is the direct analogy of the ECE Coulomb law. More generally mass density is defined by:

$$\rho_m = J^0/k. \tag{20.25}$$

The Newtonian force is:

$$\boldsymbol{F} = m\boldsymbol{g} \tag{20.26}$$

where m is the mass of an object in a gravitational acceleration $\boldsymbol{g}$. Therefore the concept of field of force in Newtonian dynamics is inferred to be:

$$\boldsymbol{F} = E_0\boldsymbol{T} \tag{20.27}$$

where:

$$E_0 = mc^2 \tag{20.28}$$

is rest energy. The Newtonian force is defined by the orbital torsion:

$$\boldsymbol{F} = E_0(T^{010}\boldsymbol{i} + T^{020}\boldsymbol{j} + T^{030}\boldsymbol{k}) \tag{20.29}$$

and the inverse square law is described by:

$$\boldsymbol{\nabla} \cdot \boldsymbol{F} = E_0 k\rho_m. \tag{20.30}$$

However, the Newtonian dynamics are controlled not only by Eqs. (20.29) and (20.30) but also by their Hodge duals:

$$\widetilde{\boldsymbol{F}} = E_0(\widetilde{T}^{010}\boldsymbol{i} + \widetilde{T}^{020}\boldsymbol{j} + \widetilde{T}^{030}\boldsymbol{k}) \tag{20.31}$$

and

$$\boldsymbol{\nabla} \cdot \widetilde{\boldsymbol{F}} = 0 \tag{20.32}$$

and this is a new feature of Newtonian dynamics given by ECE theory.

It has also been inferred [1–10] that all orbital dynamics are controlled not by the obsolete EH field equation but by the much simpler and more powerful Theorem of Orbits:

$$n(r)r = \frac{r}{m(r)} = \int dr = r + \mu. \tag{20.33}$$

In a spherically symmetric space-time this theorem gives the line element:

$$ds^2 = -\left(1 + \frac{\mu}{r}\right)c^2dt^2 + \left(1 + \frac{\mu}{r}\right)^{-1}dr^2 + r^2d\Omega^2 \tag{20.34}$$

in spherical polar co-ordinates. If the symmetric or Christoffel connection [1–11]:

$$\Gamma^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\nu\mu} \tag{20.35}$$

is assumed, the following results are obtained from the line element (20.34) by computer algebra:

$$
\left.\begin{array}{r}
R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30} = 0 \\[4pt]
\widetilde{R}^0{}_1{}^{10} + \widetilde{R}^0{}_2{}^{20} + \widetilde{R}^0{}_3{}^{30} = 0 \\[4pt]
T^{010} + T^{020} + T^{030} = 0 \\[4pt]
\widetilde{T}^{010} + \widetilde{T}^{020} + \widetilde{T}^{030} = 0.
\end{array}\right\}
\tag{20.36}
$$

Therefore the Christoffel connection of Riemann geometry without torsion cannot be used to describe orbits or dynamics in general, because this connection produces zero force and zero mass density, i.e. a universe without matter and without energy- momentum density. That is the basic problem of the EH field equation. The correct description of relativistic dynamics requires an asymmetric connection, a major advance in understanding given by ECE theory.

The spin torsion laws of relativistic dynamics are obtained by the index choice:

$$
\kappa = 1, 2, 3
\tag{20.37}
$$

and these laws are the direct analogues of the Ampère Maxwell law of ECE theory:

$$
\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{J}
\tag{20.38}
$$

and its Hodge dual law, the Faraday law of induction of ECE theory:

$$
\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}.
\tag{20.39}
$$

Here μ_0 is vacuum permeability and $\boldsymbol{J}$ is current density. In ECE theory the components appearing in these laws are as in Table 1.

So by Eq. (20.19) there are two spin torsion laws of relativistic dynamics, one being the Hodge dual of the other. The spin torsion law corresponding to the Faraday law of induction is:

$$
\boldsymbol{\nabla} \times \widetilde{\boldsymbol{T}}_1 + \frac{\partial \widetilde{\boldsymbol{T}}_2}{\partial t} = \boldsymbol{0}
\tag{20.40}
$$

where:

$$
\widetilde{\boldsymbol{T}}_1 = \widetilde{T}^{332} \boldsymbol{i} + \widetilde{T}^{113} \boldsymbol{j} + \widetilde{T}^{221} \boldsymbol{k}.
\tag{20.41}
$$

Table 20.1 Components in the ECE Laws of Electrodynamics

Law	Electric Field Strength	Magnetic Flux Density	Type
Gauss		$B^{001}, B^{002}, B^{003}$	Orbital
Coulomb	$E^{010}, E^{020}, E^{030}$		Orbital
Faraday	$E^{332}, E^{113}, E^{221}$	$B^{101}, B^{202}, B^{303}$	Spin
Ampère Maxwell	$E^{110}, E^{220}, E^{330}$	$B^{332}, B^{113}, B^{221}$	Spin

and

$$\widetilde{\boldsymbol{T}}_2 = \widetilde{T}^{101}\boldsymbol{i} + \widetilde{T}^{202}\boldsymbol{j} + \widetilde{T}^{303}\boldsymbol{k}. \tag{20.42}$$

The spin law corresponding to the Ampère Maxwell law is

$$\boldsymbol{\nabla} \times \boldsymbol{T}_1 - \frac{\partial \boldsymbol{T}_2}{\partial t} = \boldsymbol{J}_m \tag{20.43}$$

where

$$\boldsymbol{T}_1 = T^{332}\boldsymbol{i} + T^{113}\boldsymbol{j} + T^{221}\boldsymbol{k} \tag{20.44}$$

and

$$\boldsymbol{T}_2 = T^{101}\boldsymbol{i} + T^{202}\boldsymbol{j} + T^{303}\boldsymbol{k}. \tag{20.45}$$

The current density $\boldsymbol{J}_m$ is defined in Cartesian coordinates by:

$$\boldsymbol{J}_m = J_X\boldsymbol{i} + J_Y\boldsymbol{j} + J_Z\boldsymbol{k} \tag{20.46}$$

where

$$J_X = R^0{}_1{}^{01} + R^1{}_2{}^{22} + R^1{}_3{}^{31} \tag{20.47}$$

$$J_Y = R^2{}_0{}^{02} + R^2{}_1{}^{12} + R^2{}_3{}^{32} \tag{20.48}$$

$$J_Z = R^3{}_0{}^{03} + R^3{}_1{}^{13} + R^3{}_2{}^{23}. \tag{20.49}$$

Eq. (20.40) is the gravitational equivalent of the Faraday law of induction observed experimentally [1–10] in a recent European Space Agency cooperative experiment with the Austrian group of Tajmar et al.

The Euler equation of motion is given a relativistic meaning in ECE theory by considering the first Cartan structure equation:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{20.50}$$

where $\omega^a{}_b$ is the spin connection form [1–11]. The torsion form in Eq. (20.50) is defined by a frame of reference which is itself moving. If the spin connection were not considered the torsion would be due entirely to:

$$T^a(\text{static}) = d \wedge q^a \tag{20.51}$$

in a static frame. Therefore the first Cartan structure equation can be re-written as:

$$T^a = T^a(\text{static}) + \omega^a{}_b \wedge q^b. \tag{20.52}$$

The Euler equation of classical dynamics can be written [12] as:

$$\boldsymbol{T}q = \boldsymbol{T}q(\text{static}) - \boldsymbol{\Omega} \times \boldsymbol{L} \tag{20.53}$$

where $\boldsymbol{T}q$ denotes torque, $\boldsymbol{\Omega}$ denotes angular velocity and $\boldsymbol{L}$ denotes angular momentum. Thus $\boldsymbol{\Omega} \times \boldsymbol{L}$ causes the precession of for example a gyroscope. The Newtonian or inertial frame definition of torque is:

$$\boldsymbol{T}q(\text{static}) = \frac{\partial \boldsymbol{L}}{\partial t} \tag{20.54}$$

so gyroscope type precession is not present in Newtonian dynamics and was first inferred by Euler. In ECE theory it is inferred in analogy with force (Eq. (20.27)) that torque is due to spin torsion, (force as argued being due to orbital torsion). Thus by unit analysis:

$$\boldsymbol{T}q = E_0 \int \boldsymbol{T}(\text{spin}) dr. \tag{20.55}$$

In tensor notation, the first Cartan structure equation is:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu. \tag{20.56}$$

Considering the spin torsion component $\boldsymbol{T}_1$ it is seen that in vector notation that it is defined by:

$$\boldsymbol{T}_1 = \boldsymbol{\nabla} \times \boldsymbol{q} - \boldsymbol{\omega} \times \boldsymbol{q} \tag{20.57}$$

which is the direct analogy of the ECE relation:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A} \tag{20.58}$$

where $\boldsymbol{\omega}$ is the spin connection vector [1–10]. Thus it is inferred that:

$$\boldsymbol{T}q(\text{moving}) = E_0 \int \boldsymbol{T}_1 \ dr \tag{20.59}$$

$$\boldsymbol{T}q(\text{fixed}) = E_0 \int \boldsymbol{\nabla} \times \boldsymbol{q} \ dr \tag{20.60}$$

and

$$\Omega \times \boldsymbol{L} = E_0 \int \boldsymbol{\omega} \times \boldsymbol{q} \ dr. \tag{20.61}$$

Therefore the Euler equation can be derived from the first Cartan structure equation in the limit of small but non-zero spin connection. It is also inferred from EEC theory that the structure of the Euler equation is correctly covariant, it retains its form under the general coordinate transformation. Gyroscope precession is due to spinning space-time.

Therefore ECE theory has been used to infer the correctly objective equations of translational and rotational motion in relativistic dynamics. The Theorem of Orbits and the spherical symmetry of space-time describe all known orbits.

Acknowledgments

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