THE

PHOTON'S MAGNETIC FIELD

OPTICAL NMR SPECTROSCOPY

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THE PHOTON'S MAGNETIC FIELD: OPTICAL NMR SPECTROSCOPY


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# Contents

**Introduction**

vii

**Chapter 1**  
Optical NMR and ESR Spectroscopy — Equivalent Magnetic Flux Density of the Circularly Polarized Laser  
1

**Chapter 2**  
The Magnetostatic Flux Density $B_\parallel$ of the Electromagnetic Field: Development and Classical Interpretation  
37

**Chapter 3**  
The Elementary Static Magnetic Field of the Photon  
57

**Chapter 4**  
On the Experimental Measurement of the Photon’s Fundamental Static Magnetic Field Operator, $\hat{B}_\parallel$: The Optical Zeeman Effect in Atoms  
89

**Chapter 5**  
On the Experimental Detection of the Photon’s Fundamental Static Magnetic Field Operator: The Anomalous Optical Zeeman and Optical Paschen Back Effects  
110

**Chapter 6**  
The Photon’s Magnetostatic Flux Quantum: Symmetry and Wave Particle Duality, Fundamental Consequences in Physical Optics  
138

**Chapter 7**  
The Photon’s Magnetostatic Flux Quantum $\hat{B}_\parallel$: On the Absence of Faraday Induction  
171

**Chapter 8**  
The Optical Faraday Effect and Optical MCD  
183
Chapter 9  The Photon's Magnetostatic Flux Density $\hat{B}_\parallel$:  
The Inverse Faraday Effect Revisited  

Chapter 10  The Photon's Magnetostatic Flux Quantum:  
The Optical Cotton Mouton Effect  

Chapter 11  The Photon's Magnetic Flux Quantum $B_\parallel$: The  
Magnetic Nature of Antisymmetric Light Scattering  

Chapter 12  The Photon's Magnetostatic Flux Quantum:  
Forward Backward Birefringence Induced by a Laser
Chapter 2

THE MAGNETOSTATIC FLUX DENSITY \( B_\pi \) OF THE ELECTROMAGNETIC FIELD:

DEVELOPMENT AND CLASSICAL INTERPRETATION

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ABSTRACT

The classical electromagnetic field generates a static, uniform, magnetic flux density \( B_\pi \) in its axis (Z) of propagation, proportional to the antisymmetric part of the tensor \( E_i E^*_j \), (the vector quantity \( E \times E' \)) where \( E \) is the electric field strength of the plane wave. The properties of \( B_\pi \) are developed and interpreted in classical field theory. It is shown that \( B_\pi \) obeys the novel continuity equation

\[
\nabla \cdot B_\pi = -\frac{\partial U_\pi}{\partial t} = 0
\]

where \( U_\pi = -B_\pi/c \) is a quantity interpreted as a magnetostatic density of the electromagnetic plane wave. Here \( B_\pi \) is the magnetic flux density amplitude of the wave and \( c \) the speed of light. The vector potential, \( A_\pi \), of \( B_\pi \) is defined in terms of \( E \times E' \), which is shown to be proportional to the antisymmetric component of Maxwell's electromagnetic stress tensor.

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Finally, the motion of an electron in the field $B_\mu$ is shown to be a helix by solving the novel equation of motion $p + 2eA_\mu = $ constant, where $p$ is the momentum of the electron and $e$ its charge. This trajectory is shown to be equivalent to the motion of an electron in a circularly polarised plane wave.

1. INTRODUCTION

It has been demonstrated [1-7] recently that the electromagnetic plane wave generates the novel quantity $B_\mu$ which has the units of tesla, and the symmetry of uniform, divergentless, magnetostatic flux density, i.e. positive to parity inversion $\hat{P}$ and negative to motion reversal $\hat{T}$. In quantum field theory $B_\mu$ becomes [2] the novel magnetostatic flux density quantum, $\hat{B}_\mu$, a boson operator defined by

$$\hat{B}_\mu = \frac{B_0 \hat{J}}{\hbar},$$

where $\hat{J}$ is the angular momentum boson operator of the photon, $B_0$ is the magnetic flux density amplitude associated with a single photon, and $\hbar$ is the reduced Planck constant. The boson operator $\hat{B}_\mu$ is generated by each photon of the beam as it propagates linearly at the speed of light, $c$. For this reason, and from considerations of symmetry [8], there can be no electrostatic field $E_\mu$ in $X$ and $Y$ of the plane wave, and $E_\mu$ cannot be generated from $B_\mu$ by Faraday induction in
free space. However, $B_n$ interacts with matter through an interaction hamiltonian operator 
$-\mathbf{m} \cdot B_n$, where $\mathbf{m}$ is a magnetic dipole moment operator, and $B_n$ can be used either in its classical or quantum forms to describe observable phenomena such as the inverse Faraday effect \{9\}; an optical Faraday effect \{3\}; optical Zeeman effect \{4,5\}; optical Cotton Mouton effect \{6\}; and other related phenomena \{10,11\} in which light magnetizes matter. Well known phenomena of optics can also be reinterpreted in terms of $B_n$, for example antisymmetric light scattering \{12\}. The classical vector $B_n$ is proportional \{1\} to the vector cross product $\mathbf{E} \times \mathbf{E}^*$, where $\mathbf{E}$ is the usual electric field strength of an electromagnetic plane wave, and $\mathbf{E}^*$ is its complex conjugate \{13-15\}. Accordingly, in free space:

$$B_n = \frac{\mathbf{E} \times \mathbf{E}^*}{2E_0 c^2} = B_0 \mathbf{k} = \left( \frac{I_0}{\varepsilon_0 c^3} \right) \frac{3}{2} \mathbf{k} = \left( \frac{\left| \mathbf{N} \right|}{2\varepsilon_0 c^3} \right) \mathbf{k}$$

(1)

in which $\mathbf{k}$ is a unit axial vector in $Z$, the propagation axis, and where $E_0$ is the scalar amplitude of $\mathbf{E}$; $B_0 = E_0 / c$ is the scalar magnetic flux density amplitude, and $\varepsilon_0$ is the free space permittivity. Here $I_0$ is the scalar intensity in watts m$^2$ of the plane wave and $|\mathbf{N}|$ denotes the scalar magnitude of the well known Poynting vector

$$\mathbf{N} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}^*$.

(2)

This paper is concerned with the development and classical interpretation of the novel vector $B_n$ in analogy with its well known relative, $\mathbf{N}$. The latter is a flux of energy density of the plane wave, and takes meaning \{15\} only when the wave interacts with matter, at the simplest level the electronic charge. It is well known that $\mathbf{N} = 2I_0 \mathbf{n}$, where $\mathbf{n}$ in free space is the unit propagation vector in the axis, $Z$, of propagation of the plane wave. The
vectors \( \mathbf{N} \) and \( \mathbf{n} \) are negative to both \( \hat{p} \) and \( \hat{T} \), and are polar vectors, whereas \( \mathbf{B}_n \) is \( \hat{p} \) positive, \( \hat{T} \) negative, and is an axial vector. \( \mathbf{N} \) is proportional to the scalar part of the free space intensity tensor

\[
I_{ij} = \varepsilon_0 c E_i E_j^*
\]

and \( \mathbf{B}_n \) is proportional to its antisymmetric part, which in vector notation is \( \varepsilon_0 E \times E^* \).

Therefore \( \mathbf{N} \) and \( \mathbf{B}_n \) are different parts of the same tensor property. It follows that as for \( \mathbf{N} \) \{15\}, its relative \( \mathbf{B}_n \) takes meaning only when the wave interacts with matter.

Section 2 develops a novel continuity equation for \( \mathbf{B}_n \) which links it to a novel magnetic density \( U_n = -\mathbf{B}_n / c \) in the same way as \( \mathbf{N} \) is linked by a continuity equation of electromagnetic field theory to the energy density \( U \) \{15\}. Thus, \( \mathbf{N} \) and \( \mathbf{B}_n \) are vector fields and \( U \) and \( U_n \) are scalar fields. The scalar field \( U \) can also be interpreted as electromagnetic power per unit volume, and \( \mathbf{N} \) as power per unit area. In the same way \( \mathbf{B}_n \) becomes interpretable in Section 2 as magnetostatic power per unit area, and \( U_n \) as magnetostatic power per unit volume generated by a completely circularly polarised electromagnetic plane wave.

Section 3 defines the vector potential \( \mathbf{A}_n \) associated with \( \mathbf{B}_n \) starting directly from a consideration of the cross product \( \mathbf{E} \times \mathbf{E}' \), suitably scaled by the denominator \( 2E_0ci \). It is shown that a vector triple product of the type \( \mathbf{r} \times (\mathbf{E} \times \mathbf{E}') \), where \( \mathbf{r} \) is a position vector in \((X, Y, Z)\), has all the characteristics of the vector potential normally associated with a uniform,
The Magnetostatic Flux Density $B_\parallel$ of the Electromagnetic Field

divergentless, magnetostatic field. The latter is identified therefore as $B_\parallel$, and is related to $A_\parallel$ through

$$B_\parallel = \nabla \times A_\parallel.$$  \hspace{1cm} (4)

Section 4 interprets the cross product $E \times E'$ with the antisymmetric part of Maxwell's electromagnetic stress tensor \{14\}, which is part of the electromagnetic energy/momentum four tensor. Thus, $B_\parallel$ is proportional to a vorticity in the classical electromagnetic field, and this is illustrated by determining the equations of motion of an electron in $B_\parallel$ by solving the novel Lorentz equation

$$p + 2eA_\parallel = \text{constant}$$ \hspace{1cm} (5)

where $p$ is the electron's momentum and $e$ its charge. The field $B_\parallel$ drives the electron forward in a helical trajectory, with constant linear velocity in $Z$. It is shown finally that this is the same trajectory as that of the electron in a circularly polarised plane wave, obtained by solving the Lorentz equation for this case. In other words, the solutions of the Lorentz equation (5) are also solutions of the Lorentz equation of motion of the electron in a circularly polarised plane wave, showing that a plane wave can generate the characteristics of a magnetostatic flux density, $B_\parallel$, which takes meaning as it interacts with the electron, driving the latter in a helical trajectory.

The paper ends with a short discussion of the interpretation of $B_\parallel$ in the required relativistic context.
2. THE CONTINUITY EQUATION FOR $B_\mu$

Maxwell's phenomenological equations lead to the following well known \(15\) continuity equation when electromagnetic radiation interacts with matter:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \mathbf{U}}{\partial t} - \mathbf{E} \cdot \mathbf{J}^* = 0$$

(6)

where $\mathbf{J}^*$, the current density, is zero in free space. The energy density, $\mathbf{U}$, is defined in free space as the time averaged quantity:

$$U = \frac{1}{2} (\mathbf{E}^* \cdot \mathbf{B} + \mathbf{E} \cdot \mathbf{D})$$

(7)

with $\mathbf{B} = \mu_0 \mathbf{H}$ and $\mathbf{D} = \varepsilon_0 \mathbf{E}$. Here $\mu_0$ is the permeability of free space.

Clearly, in free space, the wave does no work on matter, such as electronic charge, because the density $\mathbf{E} \cdot \mathbf{J}^*$ of power lost from the fields $\mathbf{E}$ and $\mathbf{B}^*$ is zero. $\mathbf{U}$ is therefore a field energy density which takes meaning only if there is field / matter interaction.

By considering the product $\mathbf{E} \times \mathbf{E}^*$, which is proportional, as we have seen, to the antisymmetric vector part of light intensity, it can be demonstrated as follows that there exists a novel continuity equation linking $B_\mu$ and $U_\mu$. Using the vector relation:

$$\nabla \cdot (\mathbf{E} \times \mathbf{E}^*) = \mathbf{E}^* \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{E}^*)$$

(8)

and the Maxwell equations in free space:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}; \quad \nabla \times \mathbf{E}^* = -\frac{\partial \mathbf{B}^*}{\partial t}$$

(9)
The Magnetostatic Flux Density $B_n$ of the Electromagnetic Field

implies

$$\nabla \cdot (E \times E^*) = \nabla \cdot (B \times B^*) = 0 \quad (10)$$

i.e.

$$\nabla \cdot B_n = 0 \quad (11)$$

which shows that $B_n$ is uniform and divergentless. From eqns (9) and (10), we can write:

$$\nabla \cdot (E \times E^*) = 2E_0 c i \nabla \cdot B_n = -E^* \cdot \frac{\partial E}{\partial t} + \nabla \cdot \frac{\partial B^*}{\partial t} \quad (12)$$

Integration by parts of the right hand side of eqn. (12) gives the result:

$$\int E^* \cdot \frac{\partial B}{\partial t} \, dt - \int E \cdot \frac{\partial B^*}{\partial t} \, dt =$$

$$\frac{1}{2} \langle E' \cdot B - E \cdot B' \rangle = -2iE_0 B_0$$

Defining the quantity

$$U_n = \frac{1}{2E_0 c i} \left( \int E^* \cdot \frac{\partial B}{\partial t} \, dt - \int E \cdot \frac{\partial B^*}{\partial t} \\, dt \right) = -\frac{B_0}{c} \quad (14)$$

implies the result

$$\nabla \cdot U_n = -\frac{\partial U_n}{\partial t} = 0 \quad (15)$$

which is a free space continuity equation for $B_n$ · Eqn. (15) is the precise counterpart of the free space continuity equation (6) for $N$:
\[ \nabla \cdot \mathbf{N} = -\frac{\partial \mathcal{U}}{\partial t} = 0 \] (16)

Eqn. (15) is novel to this work, whereas eqn. (16) is standard in classical electrodynamics [15].

The continuity equations (15) and (16) have the same structure and must be interpreted in the same way. Thus, \( \mathbf{N} \), the Poynting vector, is, in classical electrodynamics, the flux of the density \( \mathcal{U} \) of the electromagnetic energy. Similarly, the novel \( \mathbf{B}_n \) is the flux of the magnetostatic density of the electromagnetic plane wave, a flux which is uniform, and does not vary with time. \( \mathcal{U} \) is the electromagnetic field energy density in free space (i.e. electromagnetic power per unit volume [15]), and therefore \( \mathcal{U}_n \) is the magnetostatic density in free space generated by an electromagnetic plane wave, or the magnetostatic power of the wave per unit volume occupied by that wave in free space.

Both eqns. (15) and (16) are continuity equations relating \{15\} the time rate of change of a density (the scalar fields \( \mathcal{U} \) or \( \mathcal{U}_n \)) to the divergence of a flux (the vector fields \( \mathbf{N} \) or \( \mathbf{B}_n \)). It is well established \{15\} that the notion of electromagnetic field energy density, \( \mathcal{U} \), takes meaning only when there is interaction between the field and matter. Similarly, \( \mathcal{U}_n \) takes meaning only when there is wave/matter interaction. Eqns. (15) and (16) are both statements based on the existence in free space of the light intensity tensor \( I_n \), which is hermitian \{16\}. Eqn. (16) is concerned with the scalar part of \( I_n \), and eqn (15) with its vector part, which, as we have seen, is the quantity \( \varepsilon_0 c \mathbf{E} \times \mathbf{E}^\ast \) proportional to \( \mathbf{B}_n \). The existence of \( \mathbf{B}_n \), and of eqn. (15), is an inevitable consequence of the fact that \( I_n \) is a tensor \{16\}, with a
vector (i.e. antisymmetric) component which is purely imaginary as a consequence of the hermitian nature of $I_\parallel$ \cite{16}.

It follows that the classical free space electromagnetic plane wave generates $B_\parallel$, and its associated scalar field, $U_\parallel$.

3. THE VECTOR POTENTIAL $A_\parallel$ IN FREE SPACE

Since $B_\parallel$ is a magnetostatic flux density it is assumed that there exists a vector potential $A_\parallel$ such that

$$B_\parallel = \nabla \times A_\parallel.$$  \hspace{1cm} (17)

Since $B_\parallel$ and $U_\parallel$ are well defined in free space, it follows from the assumption (17) that $A_\parallel$ would also be well defined in free space. If eqn. (17) were true, it follows that $A_\parallel$ would be a function \cite{17} of the type:

$$A_\parallel = -\frac{1}{2} \mathbf{r} \times B_\parallel$$ \hspace{1cm} (18)

where $\mathbf{r}$ is a positive coordinate vector in frame $(X, Y, Z)$. From eqn. (1):

$$A_\parallel = -\frac{1}{2} \mathbf{r} \times \left( \frac{\mathbf{E} \times \mathbf{E}^*}{2E_0c\mathbf{i}} \right)$$ \hspace{1cm} (19)

Defining

$$\mathbf{r} = X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k}$$ \hspace{1cm} (20)
and using the vector relation

\[ \mathbf{x} \times (\mathbf{E} \times \mathbf{E}^*) = \mathbf{E}(\mathbf{x} \cdot \mathbf{E}^*) - \mathbf{E}^*(\mathbf{x} \cdot \mathbf{E}) \]  

(21)

with \{17-19\}

\[ \mathbf{E} = E_0 (\mathbf{i} - i\mathbf{j}) e^{i\phi} \]  

(22)

\[ \mathbf{E}^* = E_0 (\mathbf{i} + i\mathbf{j}) e^{-i\phi} \]

where \(\phi\) is the phase \{17\} of the plane wave, we have

\[ A_\mathbf{n} = \frac{B_0}{2} (X\mathbf{j} - Y\mathbf{i}) \]  

(23)

Since

\[ |B_\mathbf{n}| = B_0 \]  

(24)

it is clear that \(B_\mathbf{n}\) and \(A_\mathbf{n}\) of eqn. (24) are related by eqn. (17).

In other words there exists a vector potential \(A_\mathbf{n}\) in free space whose curl is \(B_\mathbf{n}\), and

\[ \frac{B_0}{2} (X\mathbf{j} - Y\mathbf{i}); \]  

which is defined by the vector field \(\frac{B_0}{2} (X\mathbf{j} - Y\mathbf{i});\) a field that is generated in free space by the classical electromagnetic plane waves \(22\), both solutions of Maxwell’s equations. Note that \(B_\mathbf{n}\) and \(A_\mathbf{n}\) themselves are not plane wave solutions of Maxwell’s equations. In the same way, \(N\) is not in itself a solution of Maxwell’s equations, but is generated therefrom through a cross product \(\mathbf{E} \times \mathbf{B}^* / \mu_0\).

It has therefore been demonstrated that \(B_\mathbf{n}\) is related to a well defined \(A_\mathbf{n}\) in free space, as well as to a well defined scalar field \(U_\mathbf{n}\).
The next section considers the interaction of $B_n$ with an electron, i.e. wave/matter interaction.

4. INTERACTION OF $B_n$ WITH AN ELECTRON

From eqn. (18) of the preceding section it is clear that

$$\mathbf{A}_n = -\frac{1}{2} \mathbf{v} \times \mathbf{B}_n$$

(25)

where $\mathbf{v} = \partial r / \partial t$ is a velocity in frame $(X, Y, Z)$. Consider the interaction of $B_n$ with an electron. Since action and reaction are equal and opposite the action of $B_n$ on the electronic charge $e$ is balanced by a reaction of $e$ upon $B_n$. However, as usual in classical electrodynamics, we assume that $B_n$ is not changed greatly by the action of $e$ upon it \{13-15\} so that the Lorentz equation of motion applies. Since there is no $E_n$ present,

$$\dot{p} = e \mathbf{v} \times \mathbf{B}_n$$

(26)

where $\dot{p}$ is the momentum of the electron, and $\mathbf{v}$ its velocity $(p/m)$ in the field $B_n$. With eqn. (25) the Lorentz equation can be rewritten directly in terms of the novel vector potential $A_n$:

$$\dot{p} + 2\mathbf{A}_n \dot{e} = 0.$$ 

(27)

The equations of motion of $e$ in $B_n$ thus become \{15\} those of Coriolis acceleration:

$$\dot{v}_x = \Omega v_y; \quad \dot{v}_z = 0; \quad \dot{v}_y = -\Omega v_x.$$ 

(28)
where the parameter \( \Omega \) is defined as

\[
\Omega = \left(1 - \frac{v^2}{c^2}\right)^\frac{1}{2} \left(\frac{e}{m}\right) B_{Hz}.
\]  

(29)

The equations (28) can be rewritten \{15\} as

\[
\frac{d}{dt} (v_X + iv_Y) = -i \Omega (v_X + iv_Y) ;
\]

\[
v_X + iv_Y = a \exp(-i\alpha) .
\]

(30)

where \( a \) is complex, and defined by

\[
a = v_{oc} \exp(-i\alpha)
\]

(31)

where \( v_x \) and \( \alpha \) are real. Here

\[
v_{oc} = (v_x^2 + v_y^2)^\frac{1}{2}
\]

(32)

is the velocity of the electron in the XY plane. The trajectory of the electron in the field \( B_z \)

is therefore that of a helix

\[
X = X_0 + r \sin (\Omega t + \alpha)
\]

\[
Y = Y_0 + r \cos (\Omega t + \alpha)
\]

\[
Z = Z_0 + v_{oz} t
\]

(33)

where the radius of the helix \( r \) is defined by

\[
r = \frac{D_z}{eB_{Hz}} = \frac{m}{eB_{Hz}} v_{oc} (1 - \frac{v^2}{c^2})^\frac{1}{2}
\]

(34)

where \( p_z \) is the projection of the momentum on to the XY plane.
The Magnetostatic Flux Density $B_y$ of the Electromagnetic Field...

Note that the same result can be obtained directly from the equation

$$\dot{\mathbf{r}} = -2\mathbf{v}_\theta e$$  \hfill (35)

to give

$$\dot{v}_x = -\Omega v_y \text{ etc.}$$  \hfill (36)

We now demonstrate that the motion of an electron in a circularly polarised plane wave is precisely the same as the motion of an electron in the novel magnetic field $B_n$.

Assume that the reaction of the electron upon the circularly polarised electromagnetic field is negligible, so that the Lorentz equation again applies:

$$\dot{\mathbf{r}} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$  \hfill (37)

Here $\mathbf{E}$ and $\mathbf{B}$ are plane wave solutions of Maxwell’s equations in free space:

$$\mathbf{E} = E_0(\mathbf{i} + i\mathbf{j}) e^{-i\Phi};$$

$$\mathbf{B} = B_0(\mathbf{j} - i\mathbf{i}) e^{-i\Phi}$$  \hfill (38)

i.e. they are the usual oscillating, phase dependent, electric and magnetic field vectors of the wave. Note carefully that $\mathbf{B}$ is quite different in nature from $B_n$. The velocity of the electron in frame $(X, Y, Z)$ is

$$\mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k}.$$  \hfill (39)
Substituting eqns. (38) to (39) into the Lorentz equation (37) and using $|\mathbf{B}_n| = B_0 = E_0/c$
gives the equation of motion

\begin{align*}
\dot{v}_x &= \Omega (c - v_z) e^{-i\phi} \\
\dot{v}_y &= i\Omega (c + v_z) e^{-i\phi} \\
\dot{v}_z &= \Omega (v_x + i v_y) e^{-i\phi}.
\end{align*}

(40) \quad (41) \quad (42)

Multiplying eqn (41) by $i$ and adding to eqn. (40) gives

\[ \frac{d}{dt}(v_x + iv_y) = -2v_z\Omega e^{-i\phi} \]

(43)

If we assume a solution of the type

\[ v_z = \frac{i}{2} (v_x + iv_y) e^{i\phi} \]

(44)

separation of variables occurs

\[ \frac{d}{dt}(v_x + iv_y) = -i\Omega (v_x + iv_y) \]

(45)

\[ \frac{d}{dt} v_z = -2i\Omega v_z e^{-2i\phi} \]

(46)

Eqn. (44) is a consistent solution of eqns. (40) to (42), a solution which implies that
the time average of $v_z$ must be zero

\[ \langle v_z \rangle_t = 0 \]

(47)
because the time average of the oscillating function

\[
\langle e^{i\Phi} \rangle_t = \langle \cos \phi \rangle_t + i \langle \sin \phi \rangle_t
\]

is zero \{15\}. This is consistent with the fact that the unaveraged \(v_z\) itself is in general non-zero and complex. Eqns (42) and (46) both imply that \(\langle \hat{v}_z \rangle_t\) is also zero. Eqn. (44) is therefore a solution of eqns. (40) to (42) when both \(\langle v_z \rangle_t\) and \(\langle \hat{v}_z \rangle_t\) vanish.

Note that eqn. (45) is the same precisely as eqn. (30), derived when considering the motion of one electron in \(B_n\) and can be solved to give the trajectory of the electron in a circularly polarised plane wave. This is, from eqn. (45), a circle

\[
\begin{align*}
X &= X_0 + r \sin (\Omega t + \alpha) \\
Y &= Y_0 + r \cos (\Omega t + \alpha) \\
Z &= Z_0
\end{align*}
\]

a conclusion which is consistent with that of Landau and Lifshitz \{15\} from the relativistic Hamilton Jacobi equations of the electromagnetic field, but derived in a different way by solving the Lorentz equation assuming eqns. (38).

For an initially stationary electron, i.e. for \(v_{z0} = 0\), eqns (49) are identical with eqns. (33a) to (33c), describing the trajectory of the electron in the novel field \(B_n\). The trajectories (49) are consistent with the assumption (44), an assumption which implies that the time averages of \(v_z\) and \(\hat{v}_z\) both vanish.

In summary, the trajectory of an initially stationary electron in the field \(B_n\) is the same as that in the fields \(E\) and \(B\), an initially stationary electron moves in a circle under the
influence either of $\mathbf{B}_n$ or of $\mathbf{E}$ and $\mathbf{B}$. In the former, the velocity in $Z$ vanishes identically, in
the latter this component is zero on the average.

An observer noting this trajectory would not be able to define unambiguously the influence that causes the electron to move as it does in a circle, be this the wave or the field $\mathbf{B}_n$. The influence upon an initially stationary electron of a circularly polarised
electromagnetic plane wave is identical in all respects in the plane perpendicular to the propagation axis with the influence of $\mathbf{B}_n$ upon that electron. Therefore the motion in this plane of an electron in a circularly polarised electromagnetic plane wave can be represented exactly by a magnetostatic field $\mathbf{B}_n$, which influences an electron to move in the same trajectory. $\mathbf{B}_n$ is therefore a real, physically meaningful, influence.

Furthermore, we have shown that $\mathbf{B}_n$ has the units of tesla, is $\hat{T}$ negative, $\hat{P}$ positive, is accompanied by a well defined scalar field $U_n$, and a well defined vector potential $\mathbf{A}_n$. It follows that $\mathbf{B}_n$ has several characteristics of a magnetostatic field.

However, $\mathbf{B}_n$, is clearly not identical with an ordinary magnetostatic field, because it is a property of light. It is important to note that apparently there are no $X$ and $Y$ components of $\mathbf{E}_n$, and $\mathbf{E}_n$ cannot be generated from $\mathbf{B}_n$ by Faraday induction. If there were an $\mathbf{E}_n$ in $X$
and $Y$ an electron's trajectory in that $\mathbf{E}_n$ would be a catenary \{15\} and clearly, from eqns.
(33) and (49), this is not the case. An electric field $\mathbf{E}_n$ cannot be generated from products such as $\mathbf{E} \times \mathbf{E}'$, $\mathbf{B} \times \mathbf{B}'$, $\mathbf{E} \times \mathbf{B}'$, or $\mathbf{B} \times \mathbf{E}'$ on the grounds of fundamental $\hat{P}$ and $\hat{T}$ symmetry.
Finally, further physical interpretation can be placed upon $\mathbf{E} \times \mathbf{E}'$ by considering the well-known Maxwell stress tensor \{15\}

$$
\sigma_{\alpha\beta} = \left[ -\varepsilon_0 E_\alpha E'_\beta - \mu_0 H_\alpha H'_\beta + \frac{1}{2} \delta_{\alpha\beta} (\varepsilon_0 E_\alpha E'_\alpha + \mu_0 H_\alpha H'_\alpha) \right]
$$

which is the momentum flux density of electromagnetic radiation, and part of its energy momentum four tensor \{15\}. It is clear that the antisymmetric component of $\sigma_{\alpha\beta}$ is proportional to $\mathbf{E} \times \mathbf{E}'$ in vector notation, so that the field $\mathbf{B}_\Pi$ is also proportional to the antisymmetric part of $\sigma_{\alpha\beta}$. It is significant to note in this context that the antisymmetric part of stress in mechanics is a vorticity, with the same symmetry as angular momentum, so we deduce that $\mathbf{B}_\Pi$ is proportional to the angular momentum of classical radiation, as in our operator equation $\hat{\mathbf{B}}_\Pi = \frac{\mathbf{B}_\Pi}{\hbar}$ \{2\} of quantum field theory. Clearly, angular momentum takes meaning in the energy momentum four tensor of the electromagnetic field through the antisymmetric part of the Maxwell stress tensor, a vorticity, i.e. the antisymmetric part of the momentum tensor per unit volume.

DISCUSSION

It has been argued that the notion of $\mathbf{B}_\Pi$ is consistent with several properties of a magnetostatic field, but it must be borne in mind that $\mathbf{B}_\Pi$ is generated by a photon which travels at the speed of light. The classical theory with which we have been concerned in this
paper must come to terms with the fact that $B_n$ is relativistic in nature. In so doing \{15,8\} it becomes clear that the only relativistically invariant component of $B_n$ is that in the propagation axis, which is, of course, consistent with our eqn. (1). Furthermore, it can be shown \{8\} that there is no Faraday induction by a time modulated $B_n$, i.e. the hypothetical $E_n$ cannot be generated from $\partial B_n/\partial t$ through Faraday's Law. This is a consequence of the Lorentz transformations. Furthermore, the existence of a $B_n$ is consistent with the fact that a valid solution of the Maxwell equations in free space is

$$\mathbf{B} = B_0 (\mathbf{j} - i \mathbf{A}) e^{-i\mathbf{k} \cdot \mathbf{x}}$$

(51)

where $\mathbf{B}$ is in general a magnetostatic field such as $B_n$. It has been shown that there exists a vector potential $\mathbf{A}_n$ such that $B_n = \nabla \times \mathbf{A}_n$, and there exists a scalar field $U_n$ linked to $B_n$ by a continuity equation. Since $\mathbf{A}_n$ is defined in terms of $X$ and $Y$ coordinates it is relativistically invariant, i.e. does not change under Lorentz transformation. This is consistent with the fact that $B_n$ is defined in $Z$, and is a magnetic flux density, and so is also invariant to Lorentz transformation.

Finally, it is straightforward to deduce that

$$\mathbf{N} = \pm c \mathbf{U}_n; \quad B_n = \pm c U_n \mathbf{k}$$

(52)

where $\mathbf{n}$ is a propagation vector whose magnitude is refractive index, and $\mathbf{k}$ is a unit axial vector. The first of these equations shows that the relation between energy and momentum in an electromagnetic field is the same as that in a particle moving at the speed of light \{15\}, i.e. the photon of quantum field theory. The second equation shows that there exists a similar proportionality between the magnetic flux density $B_n$ and $U_n$. 

The Photon's Magnetic Field
CONCLUSION

The classical theory of fields has been used to develop and interpret the concepts \( E \times E' \) and \( B_u \), and it has been shown that these concepts are self-consistent and physically meaningful, for example, the motion of an electron in \( B_u \) is the same as that in \( E \) and \( B \) of a plane wave. The latter can therefore be thought of as generating a magnetostatic field.

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