Einstein – Cartan – Evans in Detail

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Contents

- Cartan geometry
- ECE field equations
- Resonant Coulomb law
- Experimental proofs of ECE theory
Riemann Geometry in General Relativity

- **Covariant derivative**

\[ D_\mu V^\nu = \frac{\partial V^\nu}{\partial x^\mu} + \Gamma^\nu_{\rho\mu} V^\rho \]

- **Christoffel symbol**:

\[ \Gamma^\mu_{\nu\rho} = \Gamma^\mu_{\rho\nu} \]

- **Line element**:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu \]

- **Metric tensor**: \( g_{\mu\nu} \)

- **Flat space**:

\[ g_{\mu\nu} \rightarrow \eta_{\mu\nu} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix} \]
Cartan Geometry

- Coordinate transformation between base manifold (basis $x^\mu$) and tangent space (basis $x^a$)

\[ V^a = q^a_\nu V^\nu \]

- $q^a_\nu$ is 4x4 matrix

- Connection to Einstein theory:

\[ g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \]
Covariant Derivative

- Covariant derivative in tangent space
  \[ D_\mu V^a = \frac{\partial V^a}{\partial x^\mu} + \omega^a_{\mu b} V^b \]
- \( \omega^a_{\mu b} \) is "spin connection"
- definition of 1- and 2-forms, exterior product
  \[
  \left( D V^a \right)_\mu = D_\mu V^a \\
  \left( d \wedge X \right)_{\mu \nu} = \partial_\mu X_\nu^a - \partial_\nu X_\mu^a \\
  \left( X^a \wedge Y^b \right)_{\mu \nu} = X_\mu^a Y_\nu^b - X_\nu^a Y_\mu^b
  \]
Cartan Structure Equations

First and second Maurer-Cartan structure equations (2-forms)

\[ T^a = d \wedge q^a + \omega^a_b \wedge q^b \]
\[ R^a_b = d \wedge \omega^a_b + \omega^a_c \wedge \omega^c_b \]

First and second Bianchi identity (2-forms)

\[ d \wedge T^a + \omega^a_b \wedge T^b = R^a_b \wedge q^b \]
\[ d \wedge R^a_b + \omega^a_c \wedge R^c_b - R^a_c \wedge \omega^c_b = 0 \]

\[ D \wedge T^a = R^a_b \wedge q^b \]
\[ D \wedge R^a_b = 0 \]
ECE Wave Equation

- **Tetrad postulate**
  - Ensures independence of physical quantities from coordinate system (metric compatibility)
    \[ Dq^a = 0 \]

- **Taking additional derivative yields**
  - (13 proofs by Evans!)
    \[ \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \]
  - or with Einstein relation \( R = -kT \)
    \[ \left( \frac{\partial^2}{\partial t^2} + kT \right) q^a = 0 \]
Hodge Dual

- Hodge dual of a tensor in 4 dimensions:
  \[ \tilde{X}_{\mu\nu} = \frac{1}{2} |g|^{1/2} \varepsilon^{\rho\sigma}_{\mu\nu} X_{\rho\sigma} \]
  - \( \varepsilon \) is Levi-Civita-Symbol
  - \( |g| \) cancels out in most cases

- Example: electromagnetic field tensor

\[
F^{\mu\nu} = \begin{pmatrix}
0 & -E^1 & -E^2 & -E^3 \\
E^1 & 0 & -cB^3 & cB^2 \\
E^2 & cB^3 & 0 & -cB^1 \\
E^3 & -cB^2 & cB^1 & 0
\end{pmatrix}
\quad
\tilde{F}^{\mu\nu} = \begin{pmatrix}
0 & -cB^1 & -cB^2 & -cB^3 \\
cB^1 & 0 & -E^3 & E^2 \\
cB^2 & E^3 & 0 & -E^1 \\
cB^3 & -E^2 & E^1 & 0
\end{pmatrix}
\]
ECE Postulates

- Electromagnetic potential is proportional to tetrad
- Electromagnetic field is proportional to torsion

\[ A^a = A^{(0)} q^a \]
\[ F^a = A^{(0)} T^a \]

- All physics is geometry
- Potential is a genuine physical quantity
ECE Field Equations

- Indexless form notation (condensed form)

\[
\begin{align*}
D \wedge F &= R \wedge A \\
D \wedge R &= 0 \\
F &= D \wedge A \\
R &= D \wedge \omega \\
\left(\square + kT\right)A &= 0
\end{align*}
\]

- Bianchi identities
- Structure equations
- Wave equation
Introduction of Current Terms

- From definitions follows

\[ d \wedge F = A^{(0)} \left( R \wedge q - \omega \wedge T \right) =: \mu_0 j \]
\[ d \wedge \widetilde{F} = A^{(0)} \left( \widetilde{R} \wedge q - \omega \wedge \widetilde{T} \right) =: \mu_0 J \]

- Maxwell-like field equations (3-forms)

\[ \left( d \wedge F^a \right)_{\mu\nu\rho} = \left( \mu_0 j^a \right)_{\mu\nu\rho} \]
\[ \left( d \wedge \widetilde{F}^a \right)_{\mu\nu\rho} = \left( \mu_0 J^a \right)_{\mu\nu\rho} \]
Tensor and Vector Notation of ECE Field Equations

- **Tensor notation**

\[
\begin{align*}
\partial_\mu F^a_{\nu\rho} + \partial_\rho F^a_{\mu\nu} + \partial_\nu F^a_{\rho\mu} &= \mu_0 \left( j^a_{\mu\nu\rho} + j^a_{\rho\mu\nu} + j^a_{\nu\rho\mu} \right) \\
\partial_\mu \tilde{F}^a_{\nu\rho} + \partial_\rho \tilde{F}^a_{\mu\nu} + \partial_\nu \tilde{F}^a_{\rho\mu} &= \mu_0 \left( J^a_{\mu\nu\rho} + J^a_{\rho\mu\nu} + J^a_{\nu\rho\mu} \right)
\end{align*}
\]

- **Vector notation**

\[
\begin{align*}
\nabla \cdot \mathbf{B}^a &= \mu_0 j^{0a} \\
\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} &= \mu_0 j^a \\
\nabla \cdot \mathbf{E}^a &= \mu_0 J^{0a} \\
\nabla \times \mathbf{B}^a - \frac{1}{c^2} \frac{\partial \mathbf{E}^a}{\partial t} &= \mu_0 J^a
\end{align*}
\]
Field Equations for the Potential

- Field equations in indexless notation
  \[ d \wedge F = \mu_0 j \]
  \[ d \wedge \tilde{F} = \mu_0 J \]

- From structure equation \( F = D^A \) follows
  \[ d \wedge F = d \wedge (d \wedge A + \omega \wedge A) = \mu_0 j \]
  \[ d \wedge \tilde{F} = d \wedge (d \wedge \tilde{A} + \omega \wedge \tilde{A}) = \mu_0 J \]
Field-Potential Relations

Rewriting in vector form:

\[
\begin{align*}
E^a &= -\frac{\partial A^a}{\partial t} - c\nabla A^0_a - c\mathbf{\omega}^{0a}_b A^b + c\mathbf{\omega}^a_b A^0_b \\
B^a &= \nabla \times A^a - \mathbf{\omega}^a_b \times A^b
\end{align*}
\]

- Maxwell-Heaviside fields augmented by spin connection terms
- Effect of spacetime torsion, general covariance
- Spin connection is source of various new effects
Electromagnetic Potential

\[ A_\mu^a = \begin{pmatrix} A_0^0 & A_0^1 & A_0^2 & A_0^3 \\ A_1^0 & A_1^1 & A_1^2 & A_1^3 \\ A_2^0 & A_2^1 & A_2^2 & A_2^3 \\ A_3^0 & A_3^1 & A_3^2 & A_3^3 \end{pmatrix} \]

3 polarization vectors of magnetic vector potential
Resonance Equations

- Rewrite field equation containing the Coulomb law
  \[ d \wedge \left( d \wedge \tilde{A} + \omega \wedge \tilde{A} \right) = \mu_0 J \]
  to form
  \[ \phi'' + \alpha \phi' + \omega_0^2 \phi = \mu_0 J^0 \]

- Equation of forced oscillation
  - for oscillatory J
  - linear in \( \Phi \)
  - coefficients may depend on space/time coordinates
Generally Covariant Coulomb Law

- Coulomb law (simplified ECE ansatz),

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon^0}, \quad \mathbf{E} = -\left(\nabla + \boldsymbol{\omega}\right)\Phi \]

- gives generalized Poisson equation:

\[ \nabla^2 \phi + \boldsymbol{\omega} \cdot \nabla \Phi + (\nabla \cdot \boldsymbol{\omega})\Phi = -\frac{\rho}{\varepsilon^0} \]

- → resonance equation for \( \Phi \)
Resonant Coulomb Potential

- Use spherical polar coordinates, only r-dependence

\[ E_r = -\left( \frac{\partial}{\partial r} \Phi + \omega_r \Phi \right) \]

- Comparison with off-resonance case gives

\[ \omega_r = \pm \frac{1}{r} \]

- Generalized Poisson equation:

\[ \frac{d^2}{dr^2} \Phi + \frac{1}{r} \frac{d}{dr} \Phi - \frac{1}{r^2} \Phi = -\frac{\rho}{\varepsilon^0} \]
Interpretation of Resonance

Where does the energy come from?

- Consider mechanical analogue:

- Spring constant
- Mass position
- Oscillating force
- Spin connection
- Electric potential
- Charge density non-consuming, powered from spacetime
Aharonov-Bohm Effect

- Role of vector potential $A$
  - In Maxwell-Heaviside theory: not relevant (re-gaugable)
  - Re-gauging: $A \rightarrow A + \nabla \phi$

Diagram:
- Electron beam
- $B \neq 0$
- Phase shift detector
- $B = 0$, $A \neq 0$ outside coil
Explanation of Aharonov-Bohm Effect

- Experimental phase shift \( \sim \) magnetic flux:
  \[
  \delta \sim \Phi = \int_B = \int d \wedge A = \int \nabla \times A
  \]
  (outer region, must be 0)

- Explanation by ECE theory:
  \[
  \delta \sim \int d \wedge A + \int \omega \wedge A = 0 + \int \omega \wedge A \neq 0
  \]
  - Effect of spin connection \( \omega \)
  - \( \Phi \neq 0 \) where classically \( d^\wedge A = 0 \)
  - Result of spinning spacetime
Polarization of Light by a Cosmological Gravitational Field

- Circular polarization is shifted to elliptical
  - Influence of gravitation on electromagnetic radiation
  - Not explainable by Maxwell-Heaviside theory or Einstein general relativity
  - Only explainable by ECE theory
ESA Experiment (2006)

- ESA's European Space and Technology Research Centre (ESTEC)
- Experimental Detection of the Gravitomagnetic London Moment
  - The paper predicts the presence of a large gravitomagnetic field within a rotating superconductor, and describes the experimental detection of this phenomenon as an extra-gravitational acceleration on the superconductor of the order of 100µg.
  - Results „are 30 orders of magnitude higher than what general relativity predicts classically“.
  - Similar finding as for Podkletnov-Experiment (in 90’s, not well reproducible)

- In 2003 Steorn (Irish company) undertook a project to develop more efficient micro generators. Early into this project the company developed certain generator configurations that appeared to be over 100% efficient. Further investigation and development has led to the company’s current technology, a technology that produces free energy. The technology is patent pending.
- Based on „non-conservative B field“
  - resonance effect or
  - clever extraction of field energy