\[ 129(1) : \text{New derivation of the Dirac Equation,} \\
\text{Letter to Pauli: Spinors and Dirac Spinors.} \]

Consider the square of the position vector in Cartesian coordinates:

\[
r^2 = (x_i + y_j + zk) \cdot (x_i + y_j + zk) \\
= x^2 i \cdot i + y^2 j \cdot j + z^2 k \cdot k \quad - (1)
\]

Eqn. (1) may be written as:

\[
r^2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = x^2 \bar{\sigma}_1 \cdot \bar{\sigma}_1 + y^2 \bar{\sigma}_2 \cdot \bar{\sigma}_2 + z^2 \bar{\sigma}_3 \cdot \bar{\sigma}_3 \quad - (2)
\]

where:

\[
\begin{align*}
\bar{\sigma}_1 &= \bar{i} \\
\bar{\sigma}_2 &= \bar{j} \\
\bar{\sigma}_3 &= \bar{k}
\end{align*}
\]

Here, the Pauli matrices are:

\[
\bar{\sigma}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \bar{\sigma}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \bar{\sigma}_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \bar{\sigma}_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad - (6)
\]

In eqns. (3) to (5) the basis elements \( \bar{i} \) and \( \bar{j} \)
are related by \( \bar{\sigma}_1 \) and so on.

Therefore, the Pauli matrices are defined:

\[
\begin{align*}
\bar{v}_0 &= \bar{\sigma}_0 \\
\bar{v}_1 &= \bar{\sigma}_1 \\
\bar{v}_2 &= \bar{\sigma}_2 \\
\bar{v}_3 &= \bar{\sigma}_3
\end{align*}
\]

- \( (6) \)
- \( (7) \)
- \( (8) \)
- \( (9) \)
2) Transpose the tetrad to obtain consideration of rest particle Dirac spinors as follows:

\[ V_0^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = u \]

\[ V_x^T = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = u \]

\[ V_y^T = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = u \]

\[ V_z^T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = u \]

The rest spinors are given by L.H. Ryder, "Quantum Field Theory (CUP, 2nd. ed. 1996)."

Therefore:

\[ u^{(1)}(0) = \frac{1}{2} \left( V_0^T + V_z^3 T \right) \]

\[ u^{(2)}(0) = \frac{1}{2} \left( V_x^T + i V_y^T \right) \]

\[ V^{(3)}(0) = \frac{1}{2} \left( V_0^T - V_z^3 T \right) \]

\[ V^{(4)}(0) = \frac{1}{2} \left( V_x^T - i V_y^T \right) \]

The complete Dirac spinors for the rest fermion are:
\[
\begin{align*}
\phi_\uparrow &= u(0) \exp \left( -\frac{imc^2 t}{\hbar} \right), \\
\phi_\downarrow &= u(0) \exp \left( \frac{imc^2 t}{\hbar} \right)
\end{align*}
\]

The Dirac spinor is therefore a combination of two Pauli matrices, which transforms according to the Lorentz transformations.

The Dirac equation is a continuity \& \& ECE wave equation:

\[
\left( \slashed{\partial} + \kappa \gamma^0 \right) \psi = 0 - (20)
\]

where:

\[
\kappa = \frac{mc}{\hbar}
\]

Therefore eqns. (18) and (19) are:

\[
\begin{align*}
\psi_\uparrow &= u(0) \exp \left( -i \omega t \right) - (22) \\
\psi_\downarrow &= u(0) \exp \left( i \omega t \right) - (23)
\end{align*}
\]

where:

\[
\omega = c \kappa \nu
\]

Eqn. (21) indicates wave particle duality.

The energy is always positive:

\[
E_n = \frac{\hbar \omega}{2} - (25)
\]

and so there is no negative energy cascade and no Dirac sea.
The ECE wave equation is:

\[(\nabla + k \tau) \psi = 0 \] - (1)

and the Dirac equation is:

\[k \tau = \kappa^2 = \left(\frac{mc}{\tau}\right)^2 \] - (2)

The d'Alembertian is:

\[0 = \gamma^\mu \gamma^- \partial_\mu \partial^- \] - (3)

where the Dirac matrices are defined by:

\[2 \gamma^\mu = \gamma^\mu + \gamma^- \gamma^\mu \] - (4)

The wave equation (1) is therefore:

\[(i \gamma^\mu \partial^- + \kappa) (i \gamma^\mu \partial^- + \kappa) \psi = 0 \] - (5)

There are two possible solutions:

\[(i \gamma^\mu \partial^- - mc) \psi = 0 \] - (6)

or

\[(i \gamma^\mu \partial^- + mc) \psi = 0 \] - (7)

with:

\[p \mu = i \gamma^\mu \partial^- \] - (8)

The Dirac equation is originally referred to as eq. (6):

\[(\gamma^\mu p_\mu - mc) \psi = 0 \] - (9)

and is a particular case of eq. (1).
2) The Dirac matrix is:
\[ \gamma^\mu = \begin{pmatrix} \gamma^0 & \gamma^1 \\ \gamma^2 & \gamma^3 \end{pmatrix} \] 

and is made up of Pauli matrices as follows:
\[ \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \] 
\[ \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \] 
\[ \gamma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \] 
\[ \gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

This means that Dirac algebra is a consequence of ECE theory.

We have:
\[ \gamma^\mu \gamma_\mu = \gamma^0 \gamma^0 - \gamma^1 \gamma^1 = 0 \] 

so eqn. (9) is:
\[ (\gamma^0 \gamma^0 - \gamma^1 \gamma^1 - mc^2) \gamma^\mu = 0 \] 
where:
\[ \gamma^\mu = \begin{pmatrix} p_0, & -p \end{pmatrix} = \begin{pmatrix} \frac{E}{c}, & -p \end{pmatrix} \]

Now write:
\[ \gamma^\mu = \begin{pmatrix} \phi_r \\ \phi_l \end{pmatrix} \]

where \( \phi_r \) and \( \phi_l \) are the Pauli spinors. The eqn. (14) becomes:
\[
\begin{bmatrix}
-mc & p_0 + \gamma \cdot p \\
p_0 - \gamma \cdot p & -mc
\end{bmatrix}
\begin{bmatrix}
\phi_R \\
\phi_L
\end{bmatrix} = 0. - (17)
\]

As usually written it stands. Note that eq. (17) is more correctly written as:

\[
\begin{bmatrix}
-mc \gamma & p_0 \gamma + \gamma \cdot p \\
p_0 \gamma - \gamma \cdot p & -mc \gamma
\end{bmatrix}
\begin{bmatrix}
\phi_R \\
\phi_L
\end{bmatrix} = 0. - (18)
\]

The Weyl Equations

These are the Dirac equation for a rest

particle whose:

\[ p_\mu = (p_0, 0) - (19) \]

So eq. (9) becomes:

\[ (\gamma ^0 p_0 - mc) V_\mu = 0 - (20) \]

and eq. (17) is:

\[
\begin{bmatrix}
-mc & p_0 \\
p_0 & -mc
\end{bmatrix}
\begin{bmatrix}
\phi_R \\
\phi_L
\end{bmatrix} = 0 - (21)
\]

i.e.

\[ p_0 \phi_L = mc \phi_R - (22) \]
\[ p_0 \phi_R = mc \phi_L - (23) \]

\[ \gamma ^0 p_0 \gamma ^a \gamma^\mu = mc \gamma ^a \gamma^\mu - (24) \]

where

\[ p_0 = \frac{\mathbf{p}}{c} + \frac{2}{c} \frac{\mathbf{d}}{dt} - (25) \]
and \( Y^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \). \hfill (26)

Now we \( x^0 Y^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \). \hfill (27)

The Weyl equation, therefore:

\[
\int \frac{da^4}{a^4} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} mc \gamma^2 & 0 \\ 0 & mc \gamma^2 \end{bmatrix} \left( \frac{mc}{a^4} \right) \gamma^4 \]

\hfill (28)

There are four solutions:

1) \( \psi_1^R = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \exp \left( -i mc^2 \frac{t}{a^2} \right) \) \hfill (29)

2) \( \psi_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \exp \left( -i mc^2 \frac{t}{a^2} \right) \) \hfill (30)

3) \( \psi_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \exp \left( -i mc^2 \frac{t}{a^2} \right) \) \hfill (31)

4) \( \psi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \exp \left( -i mc^2 \frac{t}{a^2} \right) \) \hfill (32)
5) So for eq. (28):
\[ \frac{d}{dt} \begin{pmatrix} R \\ V_i \end{pmatrix} = \frac{mc^2}{c^2} \frac{d}{dt} \begin{pmatrix} R \\ V_i \end{pmatrix} - (33) \]

Eqs. (33) and (24) are the same as eq. (22).

Eqs. (35) and (36) are the same as eq. (22).

It is seen that eqs. (29) to (32) obey eq. (1) if the case:
\[ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \frac{2}{c^2} \frac{d}{dt} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

\[ (\square + \kappa^2) \begin{pmatrix} R \\ V_i \end{pmatrix} = 0 \]

If the column vectors in eqs. (29) to (32) are rearranged as 2 x 2 matrices, for example:
\[ \begin{pmatrix} R \\ V_i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \exp \left( -\frac{ic^2}{c^2} t \right) \] - (39)

Then eq. (1) remains true. The eigenfunction of the Dirac equation is a retard.
Notation

\[ i \frac{d}{dt} \phi_R = mc^2 \phi_L \]

\[ i \frac{d}{dt} \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix} = mc^2 \begin{bmatrix} \nu_R \\ \nu_L \end{bmatrix} \]

So:

\[ \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix} = \begin{bmatrix} \nu_R \\ \nu_L \end{bmatrix} \]

Usually \( \phi \) is known as the Dirac spinor.

ECE gives much more information about the internal structure of \( \phi \).

No Negative Energy Problem

There is no negative energy problem if the Dirac equation is properly interpreted as a limit of the Fermi postulate. The correct interpretation of Eq. (3) is:

\[ \gamma^\mu p_\mu \nu_\mu = \frac{E_0}{c} \nu_\mu \]

where

\[ E_0 = mc^2 \geq 0 \]  \hspace{1cm} (41)

and

\[ p_\mu = i \hbar \frac{\partial}{\partial \mu} \]  \hspace{1cm} (42)
The Weyl equation is essentially a quantization of
\[ E_0 = mc^2 \quad - (1) \]
It is useful to write out the procedure by which the Weyl
equation is obtained from geometry. The first step is to define
a tetrad \( g^{\alpha\beta} \) to the Weyl equation:
\[ \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} = \begin{bmatrix} 0 & \gamma & \omega \\ \gamma & 0 & \nu \\ \omega & \nu & 0 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} \quad - (2) \]

This is an example of
\[ V^a = \gamma_{\mu} V^a \quad - (3) \]
The physical meaning of the tetrad is to be determined.
The next step is to use the tetrad postulate:
\[ D^a V_\alpha = 0 \quad - (4) \]
which can be rewritten as:
\[ \Box V^\alpha = R V^\alpha \quad - (5) \]
where
\[ R = - \chi \lambda (\Gamma^\alpha_{\mu\nu} W_{\mu\nu} - \Gamma^\alpha_{\mu\nu} W_{\mu\nu}) \quad - (6) \]
which can be called the third ECE.
By hypothesis, (which can be called)
\[ R = - kT \quad - (7) \]
where \( k \) is the Einstein constant in units of action.
2) per kilogram. Here \( R \) has units of \( \text{m}^{-2} \), so \( \Phi \) has units of \( \text{kg per cubic meter} \), which is mass density. 

Therefore, eq. (5) becomes:

\[
\left( \square + kT \right) a^2 = 0 \quad (8)
\]

which is the ECE wave equation.

In the limit where the Fermi field becomes that of a free fermion:

\[
kT \rightarrow \left( \frac{mc^2}{T} \right) \quad (9)
\]

then:

\[
\lambda = \frac{mc}{kT} \quad (10)
\]

is the Compton wavelength.

In this limit, eq. (8) becomes:

\[
\left( \square + \left( \frac{mc^2}{T} \right) \right) a^2 = 0 \quad (11)
\]

which is a wave equation whose solution is

\[
\left( \square + \left( \frac{mc^2}{T} \right) \right) \left[ a_{\nu 1} a_{\nu 2} \right] = 0 \quad (12)
\]

\[
\left( \square + \left( \frac{mc^2}{T} \right) \right) a_{\nu 1} a_{\nu 2} = 0 \quad (13)
\]

...
There are four wave equations in \( \psi_1, \psi_2, \psi_3, \psi_4 \). These four wave equations are
\( \psi_1, \psi_2 \) and the corresponding Dirac equation is to the wave format of the Dirac equation. This is seen by writing \( \psi \) as a column spinor, which is a Dirac spinor
\[
\begin{pmatrix}
0 + (mc^2) \\
\psi_2 \\
\psi_3 \\
\psi_4
\end{pmatrix} = 0 - (5)
\]
\[
(0 + (mc^2)) \begin{pmatrix}
\psi_1 \\
\psi_2 \\
\psi_3 \\
\psi_4
\end{pmatrix} = 0 - (6)
\]
The two Pauli spinors are therefore:
\[
\phi^R = [\psi_1, \psi_2], \quad \phi^L = [\psi_3, \psi_4]
\]
The way in which the Dirac equation inter-relates to two Pauli spinors is found by developing the Dirac equation operator.
\[ \Theta = \lambda \mu = \nu,\ \nu', - (1) \]

Here:

\[ \chi^\mu = (\chi^0, \chi^i) \]

is the Dirac matrix. This is a 4 \times 4 matrix made up of the Pauli matrices as follows:

\[ \chi^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \chi^i = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \]

where

\[ i = 1, 2, 3 \]

The four Pauli matrices are:

\[ \sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]

and the Dirac spinor is:

\[ \chi' = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \]

From eq. (11), it is seen that \( \Theta \) is made up of five column vectors.

The five matrix equations in Eqs. (17) to (21) are the Pauli matrices.

Introduce the Pauli matrices after the three space-like Pauli matrices.
5) The cyclical relations:
\[
\left[ \frac{\sigma^1}{2}, \frac{\sigma^2}{2} \right] = i \sigma^3 \quad - (23)
\]

\[
\text{at \quad cyclicum}
\]

and this is as \( \text{SU}(2) \) symmetry relation. The
\[
\text{half-integer}
\]
\[
\text{spin} \quad 1/2
\]

With these definitions we obtain the Dirac equation:
\[
\left( i \gamma^\mu \nabla - \frac{mc}{\tau} \right) \psi = 0 \quad - (24)
\]

It is seen that eq. (24) is an equation of motion.

In this notation:
\[
\gamma^\mu \psi = \gamma^0 \psi_0 + \gamma^i \psi_i \quad - (25)
\]

where:
\[
J_0 = \frac{1}{2} \frac{2}{\tau} \quad , \quad \psi_i = \frac{1}{2} \frac{2}{\tau} \quad - (26)
\]

The de-Broglie wave-particle duality is introduced as:
\[
\psi = i \frac{a}{\tau} \quad - (27)
\]

where the four-momentum
\[
\frac{p}{c} = \left( \frac{E}{c}, \frac{p}{c} \right) \quad - (28)
\]

where \( E \) is energy.
The classical Einstein energy equation is:

\[ p^\mu p_\mu = m^2 c^2 \]  \hspace{1cm} (29)

where \[ p^\mu p_\mu = E^2 - p^2 c^2 \]  \hspace{1cm} (30)

Therefore:

\[ E^2 = c^2 p^2 + m^2 c^4 \]  \hspace{1cm} (31)

The Weyl equation is obtained when:

\[ p = 0 \]  \hspace{1cm} (32)

This means that there is no particle momentum. In this case, the particle is called "the rest particle", for which:

\[ E^2 = m^2 c^4 \]  \hspace{1cm} (33)

and:

\[ \gamma^\mu d_\mu = \gamma^0 d_0 \]  \hspace{1cm} (34)

Using Eqs. (34) and (37), the Dirac equation becomes:

\[ (\gamma^\mu p_\mu - mc) \gamma^0 = 0 \]  \hspace{1cm} (35)

The Weyl equation is therefore:

\[ \gamma^0 d_0 \gamma^\mu = \frac{mc}{c} \gamma^\mu \]  \hspace{1cm} (36)

or

\[ \gamma^0 p_0 \gamma^\mu = \frac{mc}{c} \gamma^\mu \]  \hspace{1cm} (37)
1) Development of the Wave Equation

The equation may be written as:

\[ \nu \cdot E_0 \phi = mc^2 \phi \quad (1) \]

or as:

\[ i \frac{\partial \phi}{\partial t} = \left( \frac{mc^2}{\hbar} \right) \phi \quad (2) \]

Expanding out the metrics:

\[ i \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{\partial \phi}{\partial t} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left( \frac{mc^2}{\hbar} \right) \phi \quad (3) \]

These equations denote the quantization of:

\[ E_0 = mc^2 \quad (4) \]

This energy, the rest energy \( E_0 \), is considered positive.

Neglecting rest mass \( m \) is positive. The spion is:

\[ \phi' = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \quad (5) \]

The following are four solutions to eq. (3):

\[ i \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{\partial \phi}{\partial t} e^{-i \omega t} = \omega \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} e^{-i \omega t} \quad (6) \]

\[ i \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{\partial \phi}{\partial t} e^{-i \omega t} = \omega \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} e^{-i \omega t} \quad (7) \]
\[ 3. \quad i \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{d}{dt} e^{-iat} = \omega \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} e^{-iat} - (8) \]

\[ 4. \quad i \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{d}{dt} e^{-iat} = \omega \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} e^{-iat} - (9) \]

Where \[ \omega = \frac{mc^2}{\hbar} \] - (10)

It is seen that the equation interrelate different column vectors, as follows:

\[ 0. \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad 3. \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \leftrightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad - (11) \]

\[ 3. \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \leftrightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad 4. \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \leftrightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad - (11) \]

Equations (6) to (9) are:

\[ 1. \quad i \frac{d}{dt} \begin{bmatrix} \phi_L \\ 0 \\ 0 \\ 0 \end{bmatrix} = \omega \begin{bmatrix} \phi_R \\ 0 \\ 0 \\ 0 \end{bmatrix} - (12) \]

where \[ \phi_L = \phi_R = e^{-iat} - (13) \]
2) \[ \frac{d}{dt} \begin{bmatrix} \phi_L/2 \\ \phi_R/2 \end{bmatrix} = \omega \begin{bmatrix} \phi_R/2 \\ \phi_L/2 \end{bmatrix} \] 
\[ \text{where: } \phi_L/2 = \phi_R/2 = e^{-i\alpha t}. \] 

3) \[ \frac{d}{dt} \begin{bmatrix} \phi_L \\ \phi_R \\ \phi_{L1} \\ 0 \end{bmatrix} = \omega \begin{bmatrix} 0 \\ 0 \\ 0 \\ \phi_{L1} \end{bmatrix} \] 
\[ \text{where } \phi_R = \phi_{L1} = e^{-i\alpha t}. \] 

4) \[ \frac{d}{dt} \begin{bmatrix} 0 \\ \phi_R \\ \phi_L \\ 0 \end{bmatrix} = \omega \begin{bmatrix} 0 \\ 0 \\ 0 \\ \phi_L/2 \end{bmatrix} \] 
\[ \text{where } \phi_R/2 = \phi_L/2 = e^{-i\alpha t}. \] 

Now add eqns. (12), (14), (16) and (18):

\[ \frac{d}{dt} \begin{bmatrix} \phi_L \\ \phi_R \\ \phi_{L1} \\ \phi_{L2} \end{bmatrix} = \omega \begin{bmatrix} \phi_R \\ \phi_L \\ \phi_{L1} \\ \phi_{L2} \end{bmatrix} \] 
\[ \text{where } \phi_R = \phi_L. \] 

\[ \alpha \]

\[ \frac{d}{dt} \begin{bmatrix} \phi_L \\ \phi_R \end{bmatrix} = \omega \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix} \] 
\[ \text{where } \phi_R = \phi_L. \]
4) Eq. (22) means that the rest spinors of a particle are indistinguishable. The reason is that a particle at rest has no velocity. The helicity is generated only when the momentum of a particle is non-zero. There is actually nothing left of the Weyl equation to indicate right and left helicity. The electric charge does not enter into the analysis at all. Eq. (3) contains only the zeroth level matrix.

\[
\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (23)
\]

\[
\text{so} \quad i \begin{bmatrix} 0 & \sigma^0 \\ \sigma^0 & 0 \end{bmatrix} \frac{d}{dt} = \left[ \sigma^0 \sigma^0 \right] \frac{mc^2}{\hbar} \phi R - (24)
\]

\[
i.e. \quad i \begin{bmatrix} 0 & \sigma^0 \\ \sigma^0 & 0 \end{bmatrix} \frac{d}{dt} \left[ \phi R \right] = \left[ \sigma^0 \sigma^0 \right] \frac{mc^2}{\hbar} \left[ \phi^R \right] - (25)
\]

The mass \( m \) is positive throughout the analysis, and the rest energy \( E_0 \) is positive. There is no negative energy problem throughout. There is no indication of the existence of an anti-particle because there is no helicity. Electric charge does not enter into the analysis.
The Weyl spinor of a rest particle is:

\[ \psi = \begin{bmatrix} \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array} \end{bmatrix} e^{-i\alpha t} \]  \hspace{1cm} - (1)

where:

\[ \alpha = mc^2 / \hbar \]  \hspace{1cm} - (2)

Thus:

\[ \left( \partial + \left( \frac{mc^2}{\hbar} \right) \right) \psi = 0 \]  \hspace{1cm} - (3)

where

\[ \partial = - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \]  \hspace{1cm} - (4)

Eq. (3) is true if:

\[ \psi = \begin{bmatrix} \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array} \end{bmatrix} e^{-i\alpha t} \]  \hspace{1cm} - (5)

\[ = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \]  \hspace{1cm} - (6)

Now we:

\[ \begin{bmatrix} \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array} \end{bmatrix} = \sigma^0 + \sigma^1 \]  \hspace{1cm} - (7)

where

\[ \sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \]  \hspace{1cm} - (8)

our Pauli matrices.

Therefore:
The tetrad representation of the spinor of a rest particle is therefore:
\[ \psi^a = \left( \sigma^\mu \right) e^{-i\alpha t} \]  
(9)

and so:
\[ \psi^a = \left( \sigma^\mu \right) e^{-i\alpha t} \]  
(10)

\[ \psi^a = \begin{bmatrix} R & R \\ \phi R & 2 \end{bmatrix} = \left( \sigma^\mu \right) e^{-i\alpha t} \]  
(11)

Q.E.D.

The Weyl equation is a special case of:
\[ \left( \Box + k^2 \right) \psi^a = 0 \]  
(12)

The ECE wave equation.

Mathematical Results:
\[ \begin{bmatrix} \sigma^0 \sigma^0 \\ 0 \sigma^1 \end{bmatrix} = \frac{1}{2} \left( \sigma^0 \sigma^0 + \sigma^1 \right) \]  
(13)

\[ \begin{bmatrix} \sigma^0 \sigma^1 \\ 0 \sigma^0 \end{bmatrix} = \frac{1}{2} \left( \sigma^0 \sigma^1 + \sigma^0 \sigma^0 \right) \]  
(14)

\[ \begin{bmatrix} \sigma^1 \sigma^0 \\ 0 \sigma^0 \end{bmatrix} = \frac{1}{2} \left( \sigma^1 \sigma^0 + \sigma^0 \sigma^1 \right) \]  
(15)
\[ [0, 0] = \frac{1}{2} (\sigma^0 - \sigma^2) - (16) \]

We have:

\[
\begin{bmatrix}
1 & 1 \\
1 & 1
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} - (17)
\]

so eqns. (13) to (16) are tetrad / spinor.

Cartan discovered spinors in 1913 and tetrad in the early twenties.

Therefore:

\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
\sigma^0 + \sigma^3 & \sigma^1 + i\sigma^2 \\
\sigma^1 - i\sigma^2 & \sigma^0 - \sigma^3
\end{bmatrix} - (18)
\]

As shown in 129(1), the Pauli matrices are themselves tetrad.

The Dirac matrices are also tetrad because:

\[
\gamma_{\mu} = \sigma^0 \gamma_{\nu}^T = \frac{1}{2} \left( \gamma^\nu \gamma^\mu + \gamma^\mu \gamma^\nu \right) - (20)
\]

where

\[
\gamma_{\mu} = \text{diag} (1, -1, -1, -1) - (21)
\]

\[
\gamma_{\alpha\beta} = \text{diag} (1, 1, 1, 1) - (22)
\]
The type of theory is described in J. S. R. Ryder and Atkin, Fearh, Klein Gordon, Weyl and Dirac equations to Minkowski spacetime is used. The probability for current ii is correct S.T. units is:

\[ j^\mu = (\mathbf{p}, j) - (1) \]

and the continuity equation is:

\[ \partial_\mu j^\mu = 0. - (2) \]

In vector notation, eq. (2) is:

\[ \frac{1}{c} \frac{\partial \mathbf{j}}{\partial t} + \nabla \cdot \mathbf{j} = 0. - (3) \]

The quantum equivalence is:

\[ p^\mu = i\hbar j^\mu - (4) \]

A real valued current density is obtained from the eigenfunction \( \phi \) by using the sum of the function and its complex conjugate. So, for example, the charge probability is:

\[ p = \frac{i\hbar}{2mc^2} \frac{\partial \phi}{\partial t} - i\hbar \frac{\partial \phi^*}{\partial t} - (5) \]

\[ p = \frac{i\hbar}{2mc^2} \left( \phi^* \frac{\partial \phi}{\partial t} - \phi \frac{\partial \phi^*}{\partial t} \right) - (6) \]
The S. I. units of $F$ are $Tm$ so is unitless. The eigenfunction $\phi$ is complex valued and unitless.

From Eq. (6):

$$\frac{d}{dt} \left( \phi^* \frac{d\phi}{dt} - \phi \frac{d\phi^*}{dt} \right) = \frac{d\phi^*}{dt} \frac{d\phi}{dt} + \phi^* \frac{d^2\phi^*}{dt^2} - \frac{d\phi}{dt} \frac{d\phi^*}{dt} - \phi \frac{d^2\phi}{dt^2} - \phi^* \frac{d^2\phi^*}{dt^2} \tag{7}$$

Therefore:

$$\frac{d\psi}{2mc} - i \frac{\hbar}{\sqrt{2}m} \left( \phi^* \gamma \phi - \phi \gamma \phi^* \right) \tag{8}$$

We see:

$$\Box = \frac{1}{c^2} \frac{d^2}{dt^2} - \nabla^2 \tag{9}$$

The Klein-Gordon equation is:

$$\left( \Box + \left( \frac{mc}{\hbar} \right)^2 \right) \phi = 0 \tag{10}$$

for a particle without spin. Also:

$$\left( \Box + \left( \frac{mc}{\hbar} \right)^2 \right) \phi^* = 0 \tag{11}$$

The probability density of the equation is:

$$\rho = \frac{i}{2mc^2} \left( \phi^* \frac{d\phi}{dt} - \phi \frac{d\phi^*}{dt} \right) \tag{12}$$
3) For a particle at rest, a solution of eqs. (10) and (11) is:
\[ \phi = \exp\left(-\frac{\text{i} mc^2}{\hbar} t\right) \]  
\[ \phi^* = \exp\left(\frac{\text{i} mc^2}{\hbar} t\right) \]

In this case:
\[ \frac{\text{d}\phi}{\text{d}t} = -\frac{\text{i} mc^2}{\hbar} \phi, \quad -\text{(15)} \]
\[ \frac{\text{d}\phi^*}{\text{d}t} = \frac{\text{i} mc^2}{\hbar} \phi^* \quad -\text{(16)} \]

and
\[ \rho = 1 \]

There is 100% probability of finding the particle at rest.

Otherwise, the Klein-Gordon equation is a second order differential equation. It needs an initial condition on \( \phi \) and \( \text{d}\phi/\text{d}t \). So in general the probability density may be negative. In quantum field theory, this problem is circumvented by second quantization, so the Klein-Gordon equation is no longer regarded as a single particle equation. The Weyl equation for a rest particle does not have this problem as shown in the next note.

The wavefunction or spin of the Weyl equation is a real spinor. It is shown in previous notes to be derivable for Canonical geometry. The probability density is given by the equation also for a Dirac spinor. The probability density is given by the expectation value. These are geometrical objects in space-time itself.

All of the properties of space-time itself for some basic mathematical concepts are given first for ease of reference. The first concept is that of a Hermitian matrix. This is a square matrix which is not changed by taking the transpose of its complex conjugate. For example, if:

\[ A = \begin{bmatrix} 1 & 1 + i \\ 1 - i & 3 \end{bmatrix}, \]

then \( A^* = A \).

The transpose of a column vector is a row vector. For example, if:

\[ A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \]

then \( A^T = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \).

A row vector multiplied by a column vector is a scalar:

\[ \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = a_1^2 + a_2^2 + a_3^2. \]

The Dirac spinor is:

\[ \psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}. \]

and its transposed complex conjugate is:

\[ \psi^+ = \begin{bmatrix} \psi_1^* & \psi_2^* & \psi_3^* & \psi_4^* \end{bmatrix}. \]

The probability density of the Dirac and Weyl equations is:
\( \rho = \chi \phi^+ \phi \) \hfill (6)

Using the rule (3):

\[
\rho = \phi_1 \phi_1^* + \phi_2 \phi_2^* + \phi_3 \phi_3^* + \phi_4 \phi_4^* \hfill (7)
\]

This is positive definite and can therefore be interpreted as a probability in quantum mechanics. The probability density of relativistic quantum mechanics is therefore defined by Hermitian elements of Cartan geometry.

In Dirac algebra and the theory of the Dirac equation, the adjoint spinor \( \bar{\psi} \) is used. This is defined by:

\[
\bar{\psi} = \psi^+ \gamma^0 \hfill (8)
\]

where \( \gamma^0 \) is the zero \( 4 \times 4 \) time-like Dirac matrix:

\[
\gamma^0 = \begin{bmatrix}
\sigma^0 & 0 \\
0 & \sigma^0
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix} \hfill (9)
\]

where

\[
\sigma^0 = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \hfill (10)
\]

is the zero order Pauli matrix, \( \sigma^0 \) the 2 x 2 unit matrix. So

\[
\bar{\psi} = \left[ \phi_1^+ \phi_2^+ \phi_3^+ \phi_4^+ \right] \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix} \hfill (11)
\]
It follows that:

\[ \rho = \bar{\psi} Y^\dagger \psi \quad - (13) \]

The purpose of introducing the adjoint spinor \( \bar{\psi} \) is to expect a value of the expectation value of \( Y^\dagger \).

Therefor, the expectation value of \( Y \) is sometimes referred to as the density of spacetime.

The expectation value was originally introduced by ECE theory, and in the same way, must be born in the context of its meaning.

More generally, the probability four current \( j^\mu \) is the expectation value of the Dirac metric \( Y^\mu \):

\[ j^\mu = \bar{\psi} Y^\mu \psi \quad - (14) \]

This current is conserved:

\[ \partial^\nu j^\nu = 0. \quad - (15) \]

Therefore:

\[ \partial^\mu (\bar{\psi} Y^\mu \psi) = 0. \quad - (16) \]

Using the Liénard–Wiechert Theorem:

\[ \partial^\mu (\bar{\psi} Y^\mu \psi) = (\partial^\nu Y^\mu \psi) \bar{\psi} Y^\nu \psi + \bar{\psi} Y^\mu \psi \partial^\nu \bar{\psi} Y^\nu \psi \quad - (17) \]
The probability flux current is:

\[ j^\mu = \left[ \begin{array}{cccc} \phi_3 & \phi_4 & \phi_1 & \phi_2 \end{array} \right] \gamma^\mu \left[ \begin{array}{c} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{array} \right] \]  

\[ \text{(18)} \]

where:

\[ \gamma^\mu = (\gamma^0, \gamma^i) \]
\[ \gamma^0 = \left[ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} \right], \quad \gamma^i = \left[ \begin{array}{c} 0 \\ 0 \\ 0 \\ -i \end{array} \right] \]

\[ \sigma^0 = \left[ \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right], \quad \sigma^1 = \left[ \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right], \quad \sigma^2 = \left[ \begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right], \quad \sigma^3 = \left[ \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right] \]

\[ \text{(19)} \]

\[ \text{(20)} \]

Therefore:

\[ j^0 = \left[ \begin{array}{cccc} \phi_3 & \phi_4 & \phi_1 & \phi_2 \end{array} \right] \left[ \begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right] \left[ \begin{array}{c} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{array} \right] \]

\[ \text{(21)} \]

\[ j^1 = \left[ \begin{array}{cccc} \phi_3 & \phi_4 & \phi_1 & \phi_2 \end{array} \right] \left[ \begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right] \left[ \begin{array}{c} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{array} \right] \]

\[ \text{(22)} \]

\[ j_0 = \phi_1 \chi + \phi_2 \chi + \phi_3 \chi + \phi_4 \chi \]

\[ \text{(23)} \]
\[ j_1 = \phi_1 \phi_2^* + \phi_2 \phi_1^* - \phi_3 \phi_4^* - \phi_4 \phi_3^* \]  

\[ j_2 = [\phi_3^* \phi_4^* + \phi_1^* \phi_2^*] \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} - (25) \]

\[ = i \begin{bmatrix} \phi_1^* \phi_3^* + \phi_1^* \phi_2^* \\ \phi_4^* \\ -\phi_3^* \\ -\phi_2^* \end{bmatrix} \begin{bmatrix} \phi_4 \\ \phi_3 \\ \phi_2 \\ \phi_1 \end{bmatrix} \]

\[ j_2 = i \left( \phi_1 \phi_3^* - \phi_2 \phi_4^* - \phi_3 \phi_1^* + \phi_4 \phi_2^* \right) \]

\[ j_3 = [\phi_3^* \phi_4^* + \phi_1^* \phi_2^*] \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} - (27) \]

\[ = \begin{bmatrix} \phi_3^* \phi_4^* + \phi_1^* \phi_2^* \\ -\phi_3 \\ \phi_4 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} \]

\[ j_3 = \phi_1 \phi_1^* - \phi_2 \phi_2^* - \phi_3 \phi_3^* + \phi_4 \phi_4^* \]

We must now test directly whether:

\[ (\partial_\mu \phi) \gamma^\mu \phi + \bar{\phi} \gamma^\mu \partial_\mu \phi = 0 \] - (29)
Casenation of Probability Density in the Weyl Equation.

In the last note it was shown that:

$$\partial \phi \mu = (d \Phi \overline{\Phi} - \overline{\Phi} \Phi) \nabla \phi + \Phi \nabla \beta \phi - (1)$$

The continuity equation is:

$$\partial \mu = 0 \quad \therefore (2)$$

For the Weyl equation of a rest particle only the $Y^0$ matrix appears. Therefore the continuity eq. (2) is:

$$\partial \phi \mu = 0 \quad \therefore (3)$$

This is analogous to casenation of charge.

In this case, it may be checked directly as follows.

$$\partial \phi \mu = (d \Phi \overline{\Phi} - \overline{\Phi} \Phi) Y^0 \phi + \overline{\Phi} Y^0 \partial \phi \phi = 0 \quad \therefore (4)$$

We have

$$Y^0 \phi = \left[ \begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array} \right] = \left[ \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right] \quad (5)$$

and

$$\partial \phi \mu = \left[ \begin{array}{c} \partial \phi \mu_1 \\ \partial \phi \mu_2 \\ \partial \phi \mu_3 \\ \partial \phi \mu_4 \end{array} \right] \quad (6)$$

So:

$$\partial \phi \mu = \phi_1 \partial \phi \mu_1 + \phi_2 \partial \phi \mu_2 + \phi_3 \partial \phi \mu_3 + \phi_4 \partial \phi \mu_4 \quad (7)$$

Similarly:

$$Y^0 \partial \phi \phi = \left[ \begin{array}{c} \partial \phi \mu_3 \\ \partial \phi \mu_4 \\ \partial \phi \mu_5 \\ \partial \phi \mu_6 \end{array} \right] \quad (8)$$
2) \[ \Phi - \Phi_0 = \Phi_1 \Phi_1 + \Phi_2 \Phi_2 + \Phi_3 \Phi_3 + \Phi_4 \Phi_4 \]  
\[ \text{So: } \]  
\[ d \Phi^0 = (\Phi_1 \Phi_1^* + \Phi_2 \Phi_2^*) + (\Phi_3 \Phi_3^* + \Phi_4 \Phi_4^*) \]  
\[ + (\Phi_3 \Phi_3^* + \Phi_4 \Phi_4^*) + (\Phi_4 \Phi_4^* + \Phi_4 \Phi_4^*) \]  
\[- (10) \]

For the Weyl equation:
\[ \Phi_1 = \Phi_2 = \Phi_3 = \Phi_4 \]  
\[ = \exp \left( - \frac{imc^2}{\hbar} t \right) \]  
\[- (11) \]

\[ \Phi_1^* = \Phi_2^* = \Phi_3^* = \Phi_4^* \]  
\[ = \exp \left( \frac{imc^2}{\hbar} t \right) \]  
\[- (12) \]

Therefore
\[ d \Phi_1^* = \frac{i}{c} \Phi_1^* \text{ etc.} \]  
\[- (13) \]

\[ d \Phi_1 = - \frac{i}{c} \Phi_1 \text{ etc.} \]  
\[- (14) \]

So
\[ \Phi_1 \Phi_1^* + \Phi_1^* \Phi_1 = 0 \]  
\[- (15) \]

Q.E.D.
Remarks

The probability density of the Weyl equation is rigorously conserved. The spinors are rigorously defined in previous notes to pages 128. In this analysis, negative energy is rejected. The classical rest energy is always:

\[ E_0 = mc^2 \quad (16) \]

For \( \psi_1, \psi_2, \psi_3 \), and \( \psi_4 \). The wave equation is

\[ (\Box + \left( \frac{mc}{E} \right)^2) \psi = 0 \quad (17) \]

\[ (\Box + \left( \frac{mc}{E} \right)^2) \psi^* = 0. \quad (18) \]

For a rest particle:

\[ \Box = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \quad (19) \]

The Dirac equations for \( \psi \) and \( \bar{\psi} \) are:

\[ (i \gamma^\mu \partial_\mu - \frac{mc}{c^2}) \psi = 0 \quad (20) \]

and

\[ \bar{\psi} \left( i \gamma^\mu \partial_\mu + \frac{mc}{c^2} \right) = 0 \quad (21) \]

The Weyl equations are:

\[ (i \gamma^0 \partial_0 - \frac{mc}{c^2}) \psi = 0 \quad (22) \]

\[ \bar{\psi} \left( i \gamma^0 \partial_0 + \frac{mc}{c^2} \right) = 0 \quad (23) \]
If we multiply eq. (22) and eq. (23) and use

\[ \Box = \gamma^\mu \gamma_\mu \partial_\mu \partial_\mu, \quad -(24) \]

\[ \frac{1}{c^2} \frac{d^2}{dt^2} \gamma^0 \gamma^0 \partial_0 \partial_0 = -(25) \]

\[ \psi \partial^2 \psi = - \frac{m^2 c^4}{\hbar^2} \psi \quad -(26) \]

This may be written:

\[ \overline{\psi} \Box \psi = - \overline{\psi} \frac{m^2 c^4}{\hbar^2} \psi \quad -(27) \]

\[ \overline{\psi} \partial_\mu \psi = - \frac{m^2 c^4}{\hbar^2} \overline{\psi} \psi \quad -(28) \]

This gives:

\[ \overline{\psi} \frac{d^2}{dt^2} \partial_\mu \psi = - \frac{m^2 c^4}{\hbar^2} \overline{\psi} \psi \quad -(29) \]

Now we:

\[ p^\mu = i \hbar \partial_\mu \quad -(30) \]

So

\[ \frac{E^2}{c^2} = - \frac{\hbar^2}{c^2} \partial_0 \partial_0 = - \frac{\hbar^2}{c^2} \frac{d^2}{dt^2} \]

and

\[ E_0 \overline{\psi} \psi = m^2 c^4 \overline{\psi} \psi \quad -(31) \]

\[ E_0 = mc^2 \]
129(9): Cancellation of Probability Four-Current

\[ \text{ii } \text{Dirac Equation} \]

This is demonstrated by using:

\[ (i \gamma^\mu \partial_\mu - mc/t) \psi = 0 \quad - \text{(1)} \]
\[ \bar{\psi} (i \gamma^\mu \partial_\mu + mc/t) \psi = 0 \quad - \text{(2)} \]

Therefore:

\[ \gamma^\mu \partial_\mu \psi = -imc/t \psi \quad - \text{(3)} \]
\[ \gamma^\mu \partial_\mu \bar{\psi} = imc/t \bar{\psi} \quad - \text{(4)} \]

It follows that:

\[ \partial_\mu (\bar{\psi} \gamma^\mu \psi) = (\partial_\mu \bar{\psi}) \gamma^\mu \psi + \bar{\psi} \gamma^\mu (\partial_\mu \psi) \]
\[ = (\gamma^\mu \partial_\mu \bar{\psi}) \psi + \bar{\psi} (\gamma^\mu \partial_\mu \psi) \]
\[ = \frac{imc}{t} \bar{\psi} \psi - \frac{imc}{t} \bar{\psi} \psi \quad - \text{(5)} \]

\[ \text{So} \quad \partial_\mu j^\mu = 0 \]

C.E.D. \quad - \text{(6)}

Here

\[ j^\mu = \bar{\psi} \gamma^\mu \psi \quad - \text{(7)} \]
Eqn. (7) shows clearly that Dirac equation is based on geometry, because:

\[ \Box = \gamma^\alpha \gamma_\alpha \partial_\alpha \]  

(8)

The probability flux current of relativistic quantum mechanics is geometrical in origin. We now know that the origin of the equation is the relathion postulate:

\[ D_\alpha \sqrt{\alpha} = 0, \]  

(9)

which can be rewritten as:

\[ \Box \sqrt{\alpha} = \mathcal{R} \sqrt{\alpha}, \]  

(10)

The origin of Dirac spinor \( \psi \) is the Cauchy world, meaning that there is no \( \psi \) at origin. Philosophically, there is nothing about geometry that is absolutely knowable.

Therefore the Copenhagen interpretation of quantum mechanics is rejected.

Any wave equation of physics is also a Dirac equation. This is because eqn. (10) is:

\[ \gamma^\alpha \partial_\alpha \psi = \mathcal{R} \psi. \]  

(11)

Eqn. (11) gives:
which is the Dirac equation in any spacetime.

Similarly: \[ \overline{\Psi} \left( i \gamma^\mu \partial_\mu + R^{1/2} \right) \Psi = 0 \] \[ \overline{\Psi} = \left( \begin{array}{c} \overline{\psi}_0 \\ \overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3 \\ \overline{\psi}_4 \end{array} \right) \] where \( \overline{\Psi} \) is the adjoint field.

Finally, eq (6) written out in full is:

\[ \begin{align*}
&d_0 \left( \psi_0 \psi_0^* + \psi_1 \psi_1^* + \psi_2 \psi_2^* + \psi_3 \psi_3^* + \psi_4 \psi_4^* \right) \\
&+ d_1 \left( \psi_0 \psi_1^* + \psi_1 \psi_0^* - \psi_2 \psi_3^* + \psi_4 \psi_2^* \right) \\
&+ d_2 \left( \psi_0 \psi_2^* - \psi_1 \psi_3^* - \psi_3 \psi_0^* + \psi_4 \psi_1^* \right) \\
&+ d_3 \left( \psi_0 \psi_1^* - \psi_2 \psi_3^* - \psi_3 \psi_1^* + \psi_4 \psi_2^* \right) \\
= 0
\]}

More generally, the probability current is:

\[ j^\mu = \overline{\Psi} \gamma^\mu \Psi \]

\[ j^\mu = \overline{\Psi} (\partial_\mu - i \gamma^\mu) \Psi \]
The Weyl equation may be written as:

\[ i \frac{d\phi^L}{dt} = \hbar c \frac{\partial}{\partial x} \phi^R - (1) \]
\[ i \frac{d\phi^R}{dt} = \hbar c \frac{\partial}{\partial x} \phi^L - (2) \]

where \( \phi^R \) and \( \phi^L \) are the Pauli spinors. The Dirac spinor is:

\[ \psi = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \]  

The fundamental symmetry operators are: \( \hat{C}, \hat{P}, \hat{T}, \hat{CP}, \hat{CT}, \hat{PT} \) and \( \hat{CPT} \). Here \( \hat{C} \) is the charge conjugation operator:

\[ \hat{C}(e) = -e \]  

which reverses the sign of electric charge \( e \). \( \hat{P} \) is the parity operator, which has the effect:

\[ \hat{P}(\vec{r}) = -\vec{r} \]  

while \( \hat{T} \) is the time reversal operator:

\[ \hat{T}(p) = -p \]  

which reverses the momentum. The Weyl equation (1) and (2) do not involve electric charge, so automatically (2) do not involve electric charge, because they conserve \( \hat{C} \). They do not involve momentum, because they conserve \( \hat{C} \). They do not involve momentum, but under \( \hat{T} \), we have:

\[ \hat{T}(t) = -t \]  

where \( t \) is the time. The spinors of the Weyl equation
\[ \phi R = \phi R = \phi L = \phi L = \exp \left( -i \frac{mc^2}{\hbar} t \right) \quad (8) \]

We interpret the ECC theory as spins of the particles and their spins with spin. More accurately, it should be the particle with velocity. When the particle is at rest, however, its velocity is zero, because it has no momentum. From eqs. (1), (2), and (8):

\[ i \frac{d}{dt} \left( \exp \left( -i \frac{mc^2}{\hbar} t \right) \right) = \frac{mc^2}{\hbar} \exp \left( -i \frac{mc^2}{\hbar} t \right) \quad (9) \]

The application of \( \Phi \) leaves the equation unchanged, because it does not contain \( \Phi \). This is true, however, only for one sense of frame. In the Cartesian system, the frame sense of chirality is defined by:

\[ i \times j = k \quad (10) \]

But we may also have all the equations of physics written in the frame of opposite chirality or handiness:

\[ i \times j = -k \quad (11) \]

For the Pauli matrices:

\[ \left[ \frac{\sigma^1}{2}, \frac{\sigma^2}{2} \right] = i \frac{\sigma^3}{2} \quad (12) \]
\[ \left[ \frac{\sigma^1}{2}, \frac{\sigma^2}{2} \right] = -i \frac{\sigma^3}{2} \quad \text{(13)} \]

Eq. (11) is generated for Eq. (10) by:
\[ \hat{P} (k) = -k \quad \text{(14)} \]

and Eq. (13) is generated for Eq. (12) by:
\[ \hat{P} (\sigma^3) = -\sigma^3 \quad \text{(15)} \]

\[ \hat{P} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{(16)} \]

In FCC theory, it will be investigated whether Eq. (16) is separate the antiparticle and not the Dirac sea. If so, Eq. (16) is preferred by Ockham's Razor.

Reversing the sign of \( t \) in Eq. (9):
\[ -i \frac{\partial}{\partial t} \left( \exp \left( \frac{imc^2 t}{\hbar} \right) \right) = \frac{mc^2}{\hbar} \exp \left( \frac{imc^2 t}{\hbar} \right) \quad \text{(11)} \]

So the Weyl equation conserves \( \frac{\sigma^3}{\sigma^3} \), \( \frac{\sigma^1}{\sigma^1} \), \( \frac{\sigma^2}{\sigma^2} \) and \( \frac{\sigma^0}{\sigma^0} \). Therefore the Weyl equation conserves \( C, P, T \), \( CPT \) and \( CPT \). It also conserves these operators if Eq. (16) is applied. This is automatic because it is independent of the choice of group.