Note 227(1) Relativistic Quantization of Particle Scattering Theory

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www.webarchive.org.uk
www.aias.us
www.atomicprecision.com
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Consider the conservation of energy momentum:

\[ p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu \quad (1) \]

This equation can be applied to scattering an reaction theory with transmutation. Note carefully that energy momentum is always conserved by definition. The basics of the theory rest on the relativistic momentum:

\[ \mathbf{p} = \gamma m \mathbf{v} \quad (2) \]

where

\[ \gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \quad (3) \]

From (2):

\[ p^2 c^2 = \gamma^2 m^2 v^2 c^2 = \gamma^2 m^2 c^4 \left( \frac{v^2}{c^2} \right) \quad (4) \]

From Eq. (3):

\[ \frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2} \quad (5) \]

so

\[ p^2 c^2 = \gamma^2 m^2 c^4 \left( 1 - \frac{1}{\gamma^2} \right) \]

\[ = \gamma^2 m^2 c^4 - m^2 c^4 \]

\[ = E^2 - E_o^2 \quad (6) \]

so

\[ E^2 = p^2 c^2 + E_o^2 \quad (7) \]

where

\[ E = \gamma mc^2, E_o = mc^2 \quad (8) \]

The Einstein energy equation (7) is a direct consequence of the relativistic momentum (2) This method is now applied to Eq. (1) as follows:

\[ (\mathbf{p}_1 + \mathbf{p}_2) \cdot (\mathbf{p}_1 + \mathbf{p}_2) = \mathbf{p}_1 \cdot \mathbf{p}_1 + \mathbf{p}_2 \cdot \mathbf{p}_2 + 2\mathbf{p}_1 \cdot \mathbf{p}_2 \]

\[ = p_1^2 + p_2^2 + 2p_1 p_2 \cos \theta \quad (9) \]

this method introduces scattering theory. Therefore:

\[ c^2 (\mathbf{p}_1 + \mathbf{p}_2) \cdot (\mathbf{p}_1 + \mathbf{p}_2) = c^2 p_1^2 + c^2 p_2^2 + 2p_1 p_2 c^2 \cos \theta \]

\[ = E_1^2 + E_2^2 - (m_1^2 + m_2^2) c^4 + 2p_1 p_2 c^2 \cos \theta \quad (10) \]
Rearranging gives:

\[ E_1^2 + E_2^2 = c^2(p_1 + p_2) \cdot (p_1 + p_2) - 2p_1p_2c^2 \cos \theta + (m_1^2 + m_2^2)c^4 \] (11)

In this equation:

\[ p_1^2 = \frac{1}{c^2}(E_1^2 - m_1^2c^4) \] (12)

\[ p_2^2 = \frac{1}{c^2}(E_2^2 - m_2^2c^4) \] (13)

Therefore:

\[ p_1p_2 = \frac{1}{c^2}((E_1^2 - m_1^2c^4)(E_2^2 - m_2^2c^4))^{1/2} \]
\[ = \frac{1}{c^2}((E_1 - m_1c^2)(E_1 + m_1c^2)(E_2 - m_2c^2)(E_2 + m_2c^2))^{1/2} \] (14)

These equations can be expressed as:

\[ p_1^2 = (\gamma_1^2 - 1)m_1^2c^2 \] (15)

\[ p_2^2 = (\gamma_2^2 - 1)m_2^2c^2 \] (16)

From Eqs (11), (15) and (16):

\[ E_1^2 + E_2^2 = c^2(p_1 + p_2) \cdot (p_1 + p_2) \]
\[ - 2(\gamma_1^2 - 1)^{1/2}(\gamma_2^2 - 1)^{1/2}m_1m_2c^4 \cos \theta \]
\[ + (m_1^2 + m_2^2)c^4 \] (17)

The covariant formulation of this equation is:

\[ (\gamma^\mu_1 + \gamma^\mu_2)(\gamma_{\mu 1} + p_{\mu 2}) = \frac{1}{c^2}(E_1 + E_2)^2 - (p_1 + p_2) \cdot (p_1 + p_2) \] (18)

From Eqn (4):

\[ (\gamma^\mu_1 + \gamma^\mu_2)(\gamma_{\mu 1} + p_{\mu 2}) = \frac{1}{c^2}(E_1 + E_2)^2 - \left(\frac{E_1^2}{c^2} + \frac{E_2^2}{c^2} - (m_1^2 + m_2^2)\right)c^2 + 2p_1p_2 \cos \theta \]
\[ = (m_1^2 + m_2^2)c^2 + 2 \left(\frac{E_1E_2}{c^2} - p_1p_2 \cos \theta\right) \] (19)

Now note that:

\[ p_1^\mu p_{\mu 2} = \frac{E_1E_2}{c^2} - p_1 \cdot p_2 \]
\[ = \frac{E_1E_2}{c^2} - p_1p_2 \cos \theta \] (20)
From Eqns (19) and (20):

\[(p_1^\mu + p_2^\mu)(p_{\mu 1} + p_{\mu 2}) = (m_1^2 + m_2^2)c^2 + 2p_1^\mu p_{\mu 2}\]  
(21)

so

\[(p_1^\mu + p_2^\mu)(p_{\mu 1} + p_{\mu 2}) - 2p_1^\mu p_{\mu 2} = (m_1^2 + m_2^2)c^2\]  
(22)

i.e.

\[p_1^\mu p_{\mu 1} + p_2^\mu p_{\mu 2} + p_1^\mu p_{\mu 1} + p_2^\mu p_{\mu 2} - 2p_1^\mu p_{\mu 2} = (m_1^2 + m_2^2)c^2\]  
(23)

Finally use:

\[p_1^\mu p_{\mu 1} = p_2^\mu p_{\mu 2}\]  
(24)

to obtain:

\[p_1^\mu p_{\mu 1} + p_2^\mu p_{\mu 2} = (m_1^2 + m_2^2)c^2\]  
(25)

which is the sum of two Einstein energy equations:

\[p_1^\mu p_{\mu 1} = m_1^2c^2\]  
(26)

and

\[p_2^\mu p_{\mu 2} = m_2^2c^2\]  
(27)

for two free and \textit{non-interacting} particles. The equation for \textit{interacting} particles is Eqn. (19):

\[(p_1^\mu + p_2^\mu)(p_{\mu 1} + p_{\mu 2}) = \frac{(E_1 + E_2)^2}{c^2} - (p_1 + p_2) \cdot (p_1 + p_2)\]

\[= (m_1^2 + m_2^2)c^2 + 2\left(\frac{E_1 E_2}{c^2} - p_1p_2\cos\theta\right)\]  
(28)

where:

\[E_1 = \gamma_1m_1c^2\]
\[E_2 = \gamma_2m_2c^2\]
\[p_1 = (\gamma_1^2 - 1)^{1/2}m_1c\]
\[p_2 = (\gamma_2^2 - 1)^{1/2}m_2c\]  
(29)

Therefore:

\[(E_1 + E_2)^2 - c^2(p_1 + p_2) \cdot (p_1 + p_2) = (m_1^2 + m_2^2)c^4 + 2(E_1 E_2 - p_1p_2c^2\cos\theta)\]

\[= (m_1^2 + m_2^2)c^4 + 2\left(\gamma_1\gamma_2m_1m_2c^4 - ((\gamma_1^2 - 1)(\gamma_2^2 - 1))^{1/2}m_1m_2c^4\cos\theta\right)\]  
(30)

This equation is suitable for factorization into a fermion equation, and is:
(E_1 + E_2)^2 - c^2(p_1 + p_2) \cdot (p_1 \cdot p_2) = (m_1^2 + m_2^2)c^4 + 2m_1 m_2 c^4 \left( \gamma_1 \gamma_2 - (\gamma_1^2 - 1)^{1/2}(\gamma_2^2 - 1)^{1/2} \cos \theta \right)

(31)

This can be written as:

(E_1 + E_2)^2 - c^2(p_1 + p_2) \cdot (p_1 + p_2) = M_2 c^4

(32)

where:

M_2 = m_1^2 + m_2^2 + 2m_1 m_2 \left( \gamma_1 \gamma_2 - (\gamma_1^2 - 1)^{1/2}(\gamma_2^2 - 1)^{1/2} \cos \theta \right)

(33)

is a varying mass term. Finally:

R := \left( \frac{Mc}{h} \right)^2

(34)

is the R parameter of ECE theory.